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The Sovereign Constraint: Sentiment, Markets, and the Limits of Foreign Policy

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Abstract

Domestic polarization and identity politics have long been treated as background conditions for foreign-policy crises, not as objects of formal analysis in their own right. This paper develops a simple microstructure-cum-geopolitics framework in which a scalar sentiment index, capturing the intensity of a given identity cleavage, simultaneously distorts portfolio choices in sovereign bond markets and shifts the government's incentives in international crises. On the financial side, we embed a Kyle-type trading environment with rational informed investors, liquidity traders, and a class of portfolio-biased investors whose demand loads on sentiment. In equilibrium, higher sentiment makes order flow less informative, raises price impact and return volatility, and generates a political component of the sovereign spread with a well-defined sentiment beta. On the policy side, we place this financial block inside a parsimonious foreign-policy game in which the government chooses between confrontation and concession, taking as given both domestic political costs and sentiment-dependent financing conditions. We show that there exists a sentiment threshold—a "constraint" associated with the relevant identity cleavage—above which the joint political and financial cost of confrontation exceeds its geopolitical benefit, so that concession becomes strictly optimal even for governments that would otherwise prefer to stand firm. We extend the analysis to dynamic sentiment, delegated-portfolio environments, and multi-asset settings, and we show that higher delegated-portfolio shares and stronger identity tilts tighten the constraint by making spreads more sensitive to sentiment. The framework nestles naturally around classic and modern episodes—from the Kulturkampf to the Russian Civil War, from Suez to the euro-area crisis—in which domestic polarization, institutional fragility, and sovereign risk interact to shape the outer bounds of foreign policy.

JEL classification: G12, G15, E44, F34, F51, N24, N44.

Keywords: sovereign risk; market microstructure; Kyle model; identity politics; polarization; delegated portfolios; foreign policy; war and peace; political spreads.

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1 Introduction

The canonical models of foreign policy under uncertainty treat financial markets as a transparent mirror of fundamentals: bond yields summarize the government's intertemporal budget constraint, and the cost of war or confrontation is simply the shadow price of resources. In practice, however, the pricing of sovereign debt is itself shaped by domestic conflicts over identity, representation, and legitimacy. When investors tilt portfolios on the basis of religion, ethnicity, ideology, or partisan allegiance, order flow becomes contaminated by sentiment, spreads acquire an explicitly political component, and the cost of financing ambitious foreign-policy choices becomes endogenous to the very cleavages that those choices may exacerbate. This paper proposes a tractable way to formalize that feedback.

We begin from a deliberately spare microstructure block. A single sovereign bond is traded by three classes of agents: a risk-neutral informed trader who observes the bond's fundamental value, a mass of liquidity traders with exogenous orders, and a class of portfolio-biased investors whose demand responds systematically to a scalar sentiment index. The latter index is designed to capture the salience of an identity cleavage—religious, ethnic, ideological, or otherwise—and is observable to all market participants. Prices are set by a competitive market maker who observes only aggregate order flow and filters fundamentals from noisy trades. In this environment, a rise in sentiment does not merely shift the level of demand; it alters the composition and variance of order flow, reduces its informational content, raises price impact, and increases conditional return volatility. We show that these effects induce a sentiment-beta in the sovereign spread and trace out how the "political" component of yields depends on the prevalence and intensity of portfolio bias.

We then place this financial block inside a stylized foreign-policy game. At the moment of crisis, the government chooses between confrontation and concession. Confrontation promises higher geopolitical payoff but entails both higher domestic political risk and a larger financing requirement at the sentiment-dependent spread generated by the microstructure equilibrium. Concession yields lower geopolitical payoff but avoids the fiscal strain and political fragility associated with escalation. The government is thus effectively constrained by a function of sentiment: as polarization increases, domestic political costs rise and bond markets charge a higher political premium to fund confrontation. We characterize the resulting constraint formally, showing that there exists a threshold level of sentiment above which concession dominates confrontation even for governments that would strictly prefer to stand firm in calmer times. In dynamic extensions where current actions feed back into future sentiment, this threshold becomes even stricter.

The remainder of the paper elaborates and applies this mechanism. After presenting the microstructure model and its sentiment-beta representation, we extend the analysis to delegated-portfolio environments in which a growing share of sovereign debt is held through mandates that encode explicit identity tilts; in such environments, the variance contribution of biased order flow, and hence the slope of spreads with respect to sentiment, is amplified. We then embed the financial block into the foreign-policy game, derive the comparative statics of the constraint with respect to both sentiment and financing needs, and illustrate how the same logic travels across historical and contemporary settings. In each case, the aim is not to claim that domestic polarization or

identity politics fully determine sovereign risk or foreign-policy choices, but rather to show that once they enter bond markets through portfolio bias, they impose a quantitatively meaningful and conceptually sharp constraint on what even determined governments can do.

Related literature

Our paper connects four strands of research: sovereign risk and political spreads, market microstructure and information in prices, identity and politics in financial markets, and formal models of foreign policy under uncertainty.

First, we build on the large literature that links sovereign spreads to default risk, macro fundamentals, and political factors. Canonical contributions such as Eaton and Gersovitz (1981) and Cole and Kehoe (2000) model sovereign default as a dynamic incentive problem, while Arellano (2008) and Aguiar and Gopinath (2006) emphasize the role of output shocks and long-run risk. A more recent strand studies how political risk, government instability, and elections feed into sovereign yields, documenting that political events and polarization can generate sizeable spread premia beyond what fundamentals would predict.[1] In contrast to these macro-sovereign frameworks, we explicitly microfound the political component of the spread through biased order flow and price impact in a Kyle environment, and then feed this "political spread" into a foreign-policy game.

Second, our financial block is rooted in the market-microstructure tradition initiated by Kyle (1985), in which a risk-neutral market maker learns about fundamentals from aggregate order flow. The idea that prices are only partially revealing because order flow mixes informed and noise trades is central in this literature; see Glosten and Milgrom (1985) and Back (1992) for early formulations, and Oehmke and Zawadowski (2016) for a modern application to credit derivatives. Our contribution is to treat identity-driven portfolios as a systematic source of "biased noise" and to show how this affects price impact, return volatility, and the sentiment beta of the sovereign spread. In doing so, we bridge the gap between microstructure-based models of information in prices and macro/sovereign models in which spreads are typically taken as reduced-form objects.

Third, we relate to work on identity, preferences, and politics in financial markets. Political and social preferences have been shown to shape portfolio choices and asset pricing, for example in the literatures on "socially responsible" or ESG investing (e.g. Hong and Kacperczyk, 2009; Pastor and Stambaugh and Taylor, 2021) and on political cycles in stock and bond returns (e.g. Santa-Clara and Valkanov, 2003; Pastor and Veronesi, 2013). At the same time, research in political economy has emphasized the role of identity and polarization for domestic policy (e.g. Akerlof and Kranton, 2000; Glaeser et al., 2005). We borrow the idea that investors can have identity-dependent tastes for particular sovereign issuers, but we embed it in a fully specified trading game that delivers a closed-form sentiment beta in the sovereign spread and a clear mapping from identity cleavages to order-flow statistics.

[1] For example, Westphalen (2005) and Becker and Schmidt (2014) document political components in CDS and bond spreads, and Alesina and Tabellini (1990) and Battaglini and Coate (2008) highlight how political frictions shape fiscal policy and debt accumulation.

Finally, our foreign-policy block is related to formal models of war, bargaining, and audience costs under incomplete information. Classic work by Fearon (1995) and Powell (1999) explains why bargaining can fail even when war is inefficient, while Schultz (1999) and Schultz (2001) emphasize how domestic political competition and audience costs discipline leaders in international crises. Historical studies have also stressed the role of financial markets and sovereign debt in constraining foreign policy choices, for example in the context of nineteenth-century great powers and twentieth-century crises.[2] Our contribution is to provide a microfounded notion of a "sovereign constraint" that operates through identity-laden portfolio bias and sentiment-dependent spreads, and to show how this mechanism generates a sharp threshold for confrontation versus concession in foreign-policy crises.

2 Microstructure with Biased Traders and Sentiment

This section restates the core linear Kyle structure of the paper and gathers the key microstructure propositions in a compact way.

2.1 Environment

There is a single risky asset, a one-period discount bond issued by the sovereign, with payoff

$$V = \mu_V + \varepsilon,$$

where $\varepsilon \sim \mathcal{N}(0, \sigma_V^2)$ is a fundamental shock. Trading takes place at date t.

There are three classes of traders: a risk-neutral informed trader I who observes V; a continuum of portfolio-biased traders B whose demand is distorted by a scalar sentiment index ϕ_t; and liquidity traders L who trade for exogenous reasons. A competitive risk-neutral market maker observes aggregate order flow Y and sets a price P to break even in expectation.

The sentiment index $\phi_t \in \mathbb{R}$ is a scalar state variable, interpreted as the intensity of a given identity cleavage or polarization dimension. In the static microstructure block we treat ϕ_t as a parameter; its dynamics become relevant in the foreign-policy block.

The informed trader observes the fundamental V and chooses a position x_I to maximize expected profits subject to a mean–variance penalty on risk-bearing capacity. Specifically, conditional on V and price P, the informed trader solves

$$\max_{x_I} \ \Pi_I(x_I; V, P) \equiv x_I(V - P) - \frac{\gamma_I}{2} x_I^2, \tag{1}$$

with risk-aversion parameter $\gamma_I > 0$. The first-order condition is

$$(V - P) - \gamma_I x_I = 0 \quad \Rightarrow \quad x_I(V, P) = \frac{V - P}{\gamma_I}.$$

[2] See, for instance, Ferguson (2006) on the "cash nexus" between finance and empire, and Flandreau and Flores (2009) on how bond markets priced political risk in historical sovereign defaults.

We therefore set

$$\kappa_I \equiv \frac{1}{\gamma_I}, \qquad x_I(V,P) = \kappa_I(V-P). \tag{2}$$

This is the canonical Kyle specification: informed demand is linear in the mispricing $V - P$, with slope κ_I summarizing effective risk-bearing capacity.

Portfolio-biased traders are uninformed about V but condition on the sentiment index ϕ_t, which they interpret as a sufficient statistic for domestic instability and identity risk. They submit a linear demand

$$x_B(\phi_t) = \mu_B - \beta_B \phi_t + u_B, \tag{3}$$

where μ_B is a baseline position, $\beta_B > 0$ measures the sensitivity of portfolio tilt to sentiment, and $u_B \sim \mathcal{N}(0, \sigma_B^2)$ is an idiosyncratic liquidity or taste shock. This linear specification can be microfounded by assuming that biased investors maximize mean–variance utility with an identity dependent taste term.[3]

Liquidity traders submit an exogenous order

$$x_L \sim \mathcal{N}(0, \sigma_L^2), \tag{4}$$

independent of (V, ϕ_t, u_B).

The idiosyncratic term u_B captures preference or mandate shocks specific to portfolio biased investors, while x_L represents truly exogenous liquidity trades unrelated to either fundamentals or sentiment.[4]

Aggregate order flow is

$$Y = x_I + x_B + x_L. \tag{5}$$

The market maker observes Y and ϕ_t (or a noisy proxy thereof in the extension below) and sets a linear pricing rule

$$P(Y, \phi_t) = \mu_V + \lambda(\phi_t)\big(Y - \mathbb{E}[Y \mid \phi_t]\big), \tag{6}$$

with $\lambda(\phi_t) > 0$ to be determined in equilibrium.

This is the standard linear pricing rule in Kyle type environments: with Gaussian fundamentals and linear demands, the competitive risk neutral market maker sets price equal to the

[3]For instance, suppose a representative biased investor solves

$$\max_{x_B} \ \mathbb{E}[x_B V] - \tfrac{\gamma_B}{2}\,\mathrm{Var}(x_B V) - \delta(\phi_t)\, x_B, \qquad \delta(\phi_t) = \delta_0 + \delta_1 \phi_t,$$

with $\gamma_B > 0$ and $\delta_1 > 0$ capturing the disutility of holding the bond for a given identity group. The first order condition is

$$\mathbb{E}[V] - \gamma_B x_B \mathrm{Var}(V) - \delta(\phi_t) = 0 \quad \Rightarrow \quad x_B(\phi_t) = \frac{\mathbb{E}[V] - \delta_0}{\gamma_B \mathrm{Var}(V)} - \frac{\delta_1}{\gamma_B \mathrm{Var}(V)}\,\phi_t,$$

which is of the form (3) with μ_B and β_B chosen appropriately; aggregating across such investors and adding idiosyncratic shocks u_B yields the specification used.

[4]In principle one could collapse these two sources of noise into a single sentiment dependent liquidity term, but keeping them separate is convenient analytically because it allows us to distinguish between sentiment driven and purely exogenous components of order flow.

conditional expectation of V given Y and ϕ_t, and joint normality implies that this conditional expectation is linear in $Y - \mathbb{E}[Y \mid \phi_t]$.

Assumption 1 (Gaussian structure) *The triple $(V - \mu_V, u_B, x_L)$ is jointly Gaussian, with zero means for $(V - \mu_V, u_B, x_L)$, and $(V - \mu_V, u_B, x_L)$ is independent of the sentiment state ϕ_t.*

2.2 Equilibrium order flow and price impact

Using (2) and (6), the informed trader's demand can be written as

$$x_I = \kappa_I \Big(V - \mu_V - \lambda(\phi_t) \big(Y - \mathbb{E}[Y \mid \phi_t] \big) \Big).$$

Substituting x_I into the definition of Y in (5) and rearranging yields the standard linear structure of order flow with an identity-biased noise component.

Lemma 1 (Equilibrium order flow) *Under Assumption 1 and the linear pricing rule (6), aggregate order flow satisfies*

$$Y - \mathbb{E}[Y \mid \phi_t] = \alpha(\phi_t) \, (V - \mu_V) + \eta(\phi_t), \tag{7}$$

where

$$\alpha(\phi_t) \equiv \frac{\kappa_I}{1 + \kappa_I \lambda(\phi_t)}, \qquad \eta(\phi_t) \equiv \frac{u_B + x_L}{1 + \kappa_I \lambda(\phi_t)}. \tag{8}$$

Proof 1 *See Appendix A.1.*

Lemma 1 shows that, as in Kyle, order flow is a linear combination of the fundamental shock $V - \mu_V$ and a composite noise term $\eta(\phi_t)$; the only distinction is that part of this noise originates in sentiment-driven portfolio tilts.

Proposition 1 (Kyle type equilibrium with identity biased noise) *Under Assumption 1 there exists a unique linear equilibrium of the form (6) in which the price impact coefficient satisfies*

$$\lambda^(\phi_t) = \frac{\kappa_I \sigma_V^2}{\sigma_B^2 + \sigma_L^2}. \tag{9}$$

Proof 2 *See Appendix A.2.*

Intuitively, the equilibrium price impact $\lambda^(\phi_t)$ balances two forces. A higher slope κ_I makes informed order flow react more strongly to mispricing, so order flow becomes more informative about the fundamental V and the market maker is willing to move price more aggressively in response to a given order. At the same time, larger noise variances σ_B^2 and σ_L^2 make it harder

to extract the informed component from total order flow, so the market maker must average over more noise and the signal in Y is diluted.

Notice that the closed form expression of $\lambda^(\phi_t)$ in (9) has the familiar Kyle form: the numerator captures the variability of fundamentals that can in principle be revealed by informed trading, while the denominator is the total noise variance coming from biased and liquidity traders. When biased order flow becomes more volatile, or when there is more exogenous liquidity trading, the denominator increases and $\lambda^(\phi_t)$ falls, reflecting the fact that prices have to move less per unit of order flow to keep the market maker's zero profit condition satisfied.

2.3 Sentiment, noise variance, and volatility

We now link the variance of identity-driven order flow to the sentiment index ϕ_t.

Assumption 2 (Sentiment-dependent variance of biased order flow) *The variance of the idiosyncratic component of portfolio-biased order flow is increasing in $|\phi_t|$:*

$$\sigma_B^2(\phi_t) = \sigma_{B,0}^2 + \theta_B \phi_t^2, \qquad \theta_B > 0. \tag{10}$$

This assumption captures the idea that identity biased investors react more strongly and more heterogeneously when sentiment becomes more intense. As polarization rises, news and political events are interpreted through a sharper identity lens, so biased orders become more volatile and more synchronized. One way to microfound this pattern is to let the sensitivity of biased demand to individual shocks increase with $|\phi_t|$, for example by specifying $x_B(\phi_t) = \mu_B - \beta_B \phi_t + \phi_t u_B$ with u_B Gaussian; aggregating across such traders yields an effective variance of the form (10). In what follows we work directly with the reduced form specification (10), which is the minimal assumption needed to obtain the comparative statics in Proposition 2.

We normalize the risk free rate to zero and interpret the equilibrium bond yield as the one period excess return on the sovereign bond. In this normalization the sovereign spread is simply the conditional expected (excess) return on the bond,

$$s(\phi_t) \equiv \mathbb{E}[R_{t+1} \mid \phi_t],$$

where $R_{t+1} = V/P_t - 1$ is the net (excess) return between t and $t+1$.[5] We interpret $s(\phi_t)$ as containing a political component that depends on sentiment and refer to it as the (sentiment driven) sovereign spread. Our focus here is on its comparative statics with respect to ϕ_t.

Proposition 2 (Sentiment, price impact, and return volatility) *Under Assumptions 1 and 2, and for a given $\kappa_I > 0$ and $\sigma_V^2 > 0$, the following statements hold in the linear equilibrium with price impact $\lambda^*(\phi_t)$:*

[5] In a more general setting with a non zero risk free rate R_f, the sovereign spread would be defined as $\mathbb{E}[R_{t+1} - R_f \mid \phi_t]$, i.e. the excess return on sovereign debt relative to a safe return.

(i) The composite noise term variance

$$\sigma_\eta^2(\phi_t) = \frac{\sigma_B^2(\phi_t) + \sigma_L^2}{\left(1 + \kappa_I \lambda^*(\phi_t)\right)^2}$$

is strictly increasing in $|\phi_t|$.

(ii) The conditional variance of the bond return $R_{t+1} = V/P_t - 1$ *is strictly increasing in* $|\phi_t|$*:*

$$Var(R_{t+1} \mid \phi_t) \quad \text{is strictly increasing in } |\phi_t|.$$

Note that here and below, we treat ϕ_t *as a given state and compare the conditional variance of returns across different sentiment states; unconditional variance would simply average these conditional variances over the distribution of* ϕ_t*.*

(iii) Suppose the stochastic discount factor, denoted M_{t+1} *and which is formally introduced in Section 2.6, prices the variance of returns with a strictly positive and increasing risk premium. In other words, assume that there exists* $\kappa_{\text{var}} > 0$ *such that the conditional expected excess return satisfies*

$$\mathbb{E}[R_{t+1} \mid \phi_t] = \kappa_{\text{var}} \text{Var}(R_{t+1} \mid \phi_t) + \textit{(terms independent of } \phi_t\textit{)}.$$

Then the sovereign spread $s(\phi_t) = \mathbb{E}[R_{t+1} \mid \phi_t]$ *is strictly increasing in* $|\phi_t|$.

Proof 3 *See Appendix A.3.*

This proposition formalizes three related intuitions. First, as sentiment moves away from its baseline, portfolio biased traders generate more volatile and more synchronized order flow, so the composite noise term $\eta(\phi_t)$ becomes more variable and order flow is less dominated by fundamentals. Second, because the market maker still has to clear the market using Y as the sufficient statistic, prices inherit this additional noise and the conditional variance of returns rises with $|\phi_t|$. Third, if investors require compensation for bearing this sentiment driven volatility, the political component of the sovereign spread, $s(\phi_t)$, must also increase in $|\phi_t|$. In other words, more intense identity politics make bond markets both noisier and more expensive as a source of funding, not because fundamentals necessarily worsen but because it becomes harder to distinguish fundamental trades from sentiment driven ones.

2.4 Delegated portfolios and amplification of sentiment

In many modern environments, a non-trivial share of sovereign debt is held through delegated portfolios whose mandates encode explicit tilts with respect to political or identity criteria. Examples include funds that constrain exposure to issuers associated with particular regimes, sectors, or communities, as well as mandates that hard-wire "reputational" or ESG-driven screens. Within our framework, such mandates can be viewed as increasing the fraction of investors whose demand is sensitive to the sentiment index ϕ_t, thereby amplifying the variance and informational content of biased order flow.

To capture this idea parsimoniously, suppose that a fraction $\theta \in [0,1]$ of total outstanding sovereign debt is held by delegated portfolios whose mandates induce a sentiment-dependent tilt of the form

$$x_B^{\text{del}}(\phi_t) = \mu_B^{\text{del}} - \beta_B^{\text{del}}\phi_t + u_B^{\text{del}},$$

with $u_B^{\text{del}} \sim \mathcal{N}(0, \sigma^2_{B,\text{del}})$, while the remaining $(1-\theta)$ of the biased holdings are in the hands of direct investors with tilt

$$x_B^{\text{dir}}(\phi_t) = \mu_B^{\text{dir}} - \beta_B^{\text{dir}}\phi_t + u_B^{\text{dir}},$$

with $u_B^{\text{dir}} \sim \mathcal{N}(0, \sigma^2_{B,\text{dir}})$. The aggregate biased demand is then

$$x_B(\phi_t;\theta) = \theta x_B^{\text{del}}(\phi_t) + (1-\theta)x_B^{\text{dir}}(\phi_t),$$

so that the idiosyncratic component of biased order flow is

$$u_B(\theta) = \theta u_B^{\text{del}} + (1-\theta)u_B^{\text{dir}}.$$

Assumption 3 (Delegated-portfolio share and biased noise) *The idiosyncratic shocks u_B^{del} and u_B^{dir} are independent and Gaussian with variances $\sigma^2_{B,del}$ and $\sigma^2_{B,dir}$, respectively, and independent of (V, ϕ_t, x_L). The effective variance of biased order flow can then be written as*

$$\sigma_B^2(\theta) = (1-\theta)^2\sigma^2_{B,dir} + \theta^2\sigma^2_{B,del}. \tag{11}$$

Under Assumption 3, the Kyle equilibrium of Proposition 1 remains valid with σ_B^2 replaced by $\sigma_B^2(\theta)$. Delegated portfolios increase the relative importance of biased order flow whenever $\sigma^2_{B,\text{del}} > \sigma^2_{B,\text{dir}}$ for $\theta > \theta_T$ a natural case if mandates induce stronger and more synchronized responses to sentiment than direct holdings.[6]

Proposition 3 (Delegation amplifies the sentiment constraint) *Suppose Assumptions 1, 2, and 3 hold, and let $\lambda^*(\phi_t;\theta)$, $\sigma_\eta^2(\phi_t;\theta)$, and $s(\phi_t;\theta)$ denote the equilibrium price impact, composite noise variance, and political component of the sovereign spread, respectively, in an economy with delegated-portfolio share θ. Then:*

If $\sigma^2_{B,del} > \sigma^2_{B,dir}$ and $\theta > \theta_T$, the composite noise variance and the political spread are both strictly increasing in θ for any fixed $\phi_t \neq 0$:

$$\frac{\partial \sigma_\eta^2(\phi_t;\theta)}{\partial \theta} > 0, \qquad \frac{\partial s(\phi_t;\theta)}{\partial \theta} > 0.$$

Proof 4 *See Appendix A.4.*

[6] For $\theta \leq \theta_T$ a classic deversification effect applies and $\sigma_B^2(\theta)$ is then decreasing in θ. Note however, that $\sigma_B^2(\theta)$ would be strictly increasing in θ whenever $\sigma^2_{B,\text{del}} > \sigma^2_{B,\text{dir}}$ if the idiosyncratic component of delegated portfolios (u_B^{del}) were perfectly positively correlated with the idiosyncratic component of biased investors (u_B^{dir}).

We will return to the implications of higher delegated portfolio shares for the foreign policy constraint in Section 3, after defining the net advantage of confrontation function $D(\phi;\theta)$ and the corresponding sentiment constraint threshold $\bar{\phi}(\theta)$ below which confrontation is preferred.

2.5 Incomplete revelation of sentiment

In many empirical contexts the market maker has only noisy information about the level of polarization or identity sentiment, over and above what can be inferred from order flow. In this extension the sentiment index ϕ_t is a latent state observed only imperfectly through two noisy channels: the biased component of aggregate order flow Y and the macro type signal $\tilde{\phi}_t = \phi_t + \xi_t$, with $\xi_t \sim \mathcal{N}(0, \sigma_\xi^2)$ independent of (V, u_B, x_L).

The competitive market maker is informationally efficient in the usual sense: it conditions on all available signals $(Y, \tilde{\phi}_t)$ when setting prices. Nevertheless, because both signals contain independent noise components, the conditional distribution of ϕ_t given market data remains diffuse, so sentiment cannot be inferred without error from observed prices and order flow. The following proposition makes this partial revelation explicit.

Proposition 4 (Partial revelation of sentiment) *In the extended setting with noisy sentiment signal $\tilde{\phi}_t$, the linear equilibrium price can be written as*

$$P - \mu_V = a_V Y + a_\phi \tilde{\phi}_t,$$

with (a_V, a_ϕ) determined by the covariance structure of $(V, Y, \tilde{\phi}_t)$. The coefficient a_ϕ is non-zero whenever portfolio-biased order flow depends on ϕ_t, so that prices load on sentiment even after conditioning on Y. However, unless $\sigma_\xi^2 = 0$, the conditional variance of sentiment given (P, Y) remains strictly positive:

$$Var(\phi_t \mid P, Y) > 0,$$

so sentiment is only partially revealed.

Proof 5 *See Appendix A.5.*

The result is best understood as a filtering statement rather than a failure of market efficiency. The sentiment index ϕ_t is an unobserved state, and both Y and $\tilde{\phi}_t$ are noisy signals of that state: order flow mixes sentiment driven trades with liquidity shocks, and $\tilde{\phi}_t$ itself is contaminated by the noise ξ_t. In a Gaussian environment the competitive market maker uses all available information in $(Y, \tilde{\phi}_t)$ to set prices, so the price is informationally efficient conditionally on these signals. However, as long as the sentiment signal is noisy ($\sigma_\xi^2 > 0$) and order flow is subject to independent shocks, no linear combination of (P, Y) can recover ϕ_t exactly, and the conditional variance $\mathrm{Var}(\phi_t \mid P, Y)$ remains strictly positive. The proposition therefore reflects incomplete information about a latent state, not a violation of the efficient markets hypothesis.

2.6 Sentiment beta in the sovereign spread

To connect the microstructure block to risk premia, we embed the pricing kernel in a simple affine factor structure, in the spirit of standard stochastic discount factor approaches to asset pricing. In a dynamic CAPM or SDF representation, the beta of an asset with respect to a factor is defined, for a one period of time, as the regression coefficient of the asset's excess return on the innovation in that factor between t and $t+1$. To implement this idea for sentiment, we therefore need a simple dynamic law of motion for the sentiment index and we work with the usual AR(1) specification below.

Our notion of a sentiment beta follows the standard dynamic CAPM logic: the beta of an asset with respect to a factor is defined as the regression coefficient of the asset's excess return on the one-period innovation in that factor between t and $t+1$. In our case, the factor is the sentiment index ϕ_t, so the relevant "innovation" is simply the unexpected change in sentiment over one period.

Let sentiment follow a stationary AR(1) process

$$\phi_{t+1} = \rho\phi_t + \zeta_{t+1}, \qquad \zeta_{t+1} \sim \mathcal{N}(0, \sigma_\zeta^2), \quad |\rho| < 1,$$

and define the one-period sentiment innovation as

$$\Delta\phi_{t+1} \equiv \phi_{t+1} - \phi_t = (\rho - 1)\phi_t + \zeta_{t+1}.$$

This $\Delta\phi_{t+1}$ is the new information about sentiment that arrives between t and $t+1$, in the same way that factor innovations enter dynamic CAPM models. We let the stochastic discount factor M_{t+1} load on sentiment innovations and other priced factors $(F_{k,t+1})_k$:

$$M_{t+1} = m_0 - m_1\Delta\phi_{t+1} - \sum_k m_k F_{k,t+1}, \qquad m_1 > 0.$$

Proposition 5 (Sentiment beta representation of the sovereign spread) *In the dynamic environment described above, the sovereign spread satisfies*

$$s(\phi_t) = \kappa_\phi\beta_{\phi,t} + \sum_k \kappa_k\beta_{k,t},$$

where

$$\beta_{\phi,t} = \frac{Cov(R_{t+1}, \Delta\phi_{t+1} \mid \phi_t)}{Var(\Delta\phi_{t+1})}$$

is the sentiment beta of the sovereign return. This $\beta_{\phi,t}$ is the usual regression coefficient of the sovereign return on the sentiment innovation $\Delta\phi_{t+1}$, so the representation above is a direct analogue of a dynamic CAPM beta decomposition with sentiment as an additional factor. Moreover,

$$\kappa_\phi = \frac{m_1\, Var(\Delta\phi_{t+1})}{\mathbb{E}[M_{t+1}]}, \qquad \kappa_k = \frac{m_k\, Var(F_{k,t+1})}{\mathbb{E}[M_{t+1}]}$$

are the corresponding prices of risk. The sentiment beta is strictly positive whenever price

impact responds to ϕ_t as in Proposition 2 and order flow loads on sentiment-dependent noise.

In particular, $\Delta\phi_{t+1}$ is not a new object but simply the one-period change in the sentiment index; calling it an "innovation" reflects the fact that, in dynamic pricing models, factor betas are always defined with respect to unexpected changes in the underlying state rather than its level.

Proof 6 *See Appendix A.6.*

From a technical point of view, nothing in this beta representation is exotic: it follows directly from the standard SDF logic used in dynamic CAPM models, with sentiment treated as an additional factor. What is specific to our setting is that the sentiment factor enters returns through the Kyle microstructure block, via the dependence of price impact and order flow variance on ϕ_t, so the sentiment beta $\beta_{\phi,t}$ has a clear structural interpretation in terms of biased trading and information in prices.

2.7 A multi-asset extension and common sentiment factors

The sentiment-beta representation in Proposition 5 extends naturally to a setting in which several sovereigns trade simultaneously and share common sentiment factors. This multi-asset extension is particularly useful for thinking about episodes such as euro-area crises, in which domestic polarization in one country generates cross-border sovereign-risk spillovers through integrated markets.

Consider n sovereign discount bonds, indexed by $i = 1, \dots, n$, each with payoff

$$V_i = \mu_i + \varepsilon_i, \qquad \varepsilon = (\varepsilon_1, \dots, \varepsilon_n)' \sim \mathcal{N}(0, \Sigma_V),$$

and let $R_{i,t+1}$ denote the one-period excess return on bond i. Let ϕ_t be a $K \times 1$ vector of sentiment factors,

$$\phi_t = (\phi_{1t}, \dots, \phi_{Kt})',$$

evolving according to a VAR(1)

$$\phi_{t+1} = \Phi\phi_t + \zeta_{t+1}, \qquad \zeta_{t+1} \sim \mathcal{N}(0, \Sigma_\zeta),$$

with Φ a $K \times K$ matrix and Σ_η positive definite.

On the trading side, suppose that for each bond i there is a vector of portfolio-biased investor types whose net demand depends linearly on ϕ_t, so that aggregate biased order flow across the n bonds can be written as

$$u_B(\phi_t) = \mu_B - B\phi_t + \upsilon_B,$$

where μ_B is an $n \times 1$ vector, B is an $n \times K$ loading matrix, and $\upsilon_B \sim \mathcal{N}(0, \Sigma_B)$ is an $n \times 1$ vector of idiosyncratic liquidity shocks. Liquidity traders contribute an additional $n \times 1$ order vector $x_L \sim \mathcal{N}(0, \Sigma_L)$. An informed trader (or vector of informed traders) observes the vector

of fundamentals $V = (V_1, \ldots, V_n)'$; prices are set by a competitive market maker who observes only aggregate order flow and (possibly noisy) signals of ϕ_t.

Under standard linear-Gaussian assumptions analogous to Assumption 1, the Kyle equilibrium generalizes to a matrix price-impact rule

$$P_t - \mu = \Lambda(\phi_t)\big(Y_t - \mathbb{E}[Y_t \mid \phi_t]\big),$$

where P_t and Y_t are $n \times 1$ vectors of prices and order flow, and $\Lambda(\phi_t)$ is an $n \times n$ matrix of impact coefficients. In this environment, the sentiment factors enter returns through their effect on $\Lambda(\phi_t)$ and on the variance and composition of order flows.

To characterize the resulting sovereign-spread structure, we embed this multi-asset microstructure block in a factor-pricing environment with a stochastic discount factor M_{t+1} that loads on both sentiment innovations and other priced factors.

Proposition 6 (Multi-country sentiment-beta representation) *Let ϕ_t be a $K \times 1$ sentiment vector evolving as above, and let the stochastic discount factor be*

$$M_{t+1} = m_0 - b'\Delta\phi_{t+1} - \sum_{\ell=1}^{L} c_\ell F_{\ell,t+1},$$

where $\Delta\phi_{t+1} = \phi_{t+1} - \phi_t$, b is a $K \times 1$ vector of sentiment prices of risk, the $F_{\ell,t+1}$ are additional priced factors with scalar prices of risk c_ℓ, and all shocks are jointly Gaussian. Denote the political component of the spread on bond i by $s_i(\phi_t)$ and its sentiment beta by

$$\beta_{\phi,i,t} \equiv \Sigma_\eta^{-1}\,\mathrm{Cov}\big(R_{i,t+1}, \Delta\phi_{t+1} \mid \phi_t\big).$$

The vector $\beta_{\phi,i,t}$ therefore collects the K regression coefficients of $R_{i,t+1}$ on the components of $\Delta\phi_{t+1}$ and generalizes the notion of a CAPM beta to a multi factor sentiment setting.

Then the vector of political spreads $s(\phi_t) = (s_1(\phi_t), \ldots, s_n(\phi_t))'$ admits the representation

$$s_i(\phi_t) = \kappa_\phi'\beta_{\phi,i,t} + \sum_{\ell=1}^{L} \kappa_\ell \beta_{\ell,i,t}, \qquad i = 1, \ldots, n, \tag{12}$$

where

$$\kappa_\phi = \frac{\Sigma_\eta b}{\mathbb{E}[M_{t+1}]}, \qquad \beta_{\ell,i,t} = \frac{\mathrm{Cov}(R_{i,t+1}, F_{\ell,t+1} \mid \phi_t)}{\mathrm{Var}(F_{\ell,t+1})}, \qquad \kappa_\ell = \frac{c_\ell \mathrm{Var}(F_{\ell,t+1})}{\mathbb{E}[M_{t+1}]}.$$

Moreover, whenever the multi-asset Kyle equilibrium satisfies $\partial\Lambda(\phi_t)/\partial\phi_{k,t} \neq 0$ for some k and bond i loads on the corresponding sentiment-dependent order-flow component, the k-th element of $\beta_{\phi,i,t}$ is strictly positive.

Proof 7 *See Appendix A.7.*

Proposition 6 formalizes the idea that, in a multi-country setting, each sovereign spread can be decomposed into a set of sentiment betas with respect to common and idiosyncratic identity

factors, plus exposure to non-sentiment risk factors. The common sentiment factors generate cross-sectional co-movements in spreads even in the absence of shared fundamentals, a feature that becomes central in applications such as the euro-area crisis and other episodes of regional polarization.

2.8 Two additional comparative-statics propositions

The existing results can be complemented with two simple comparative-statics propositions that formalize how the identity-biased component of order flow shapes the informativeness of prices and the slope of the spread in ϕ_t.

From Assumption 2, the variance of biased order flow becomes $\sigma_B^2(\phi_t) = \sigma_{B,0}^2 + \theta_B \phi_t^2$, so the equilibrium price impact $\lambda^*(\phi_t)$ in (9) is a smooth function of ϕ_t through $\sigma_B^2(\phi_t)$.

Proposition 7 (Monotone loss of informativeness in sentiment) *Under Assumptions 1 and 2, define the R^2 of price with respect to fundamentals as*

$$R^2(\phi_t) \equiv \frac{Cov(P_t, V)^2}{Var(P_t)\ Var(V)}.$$

Then $R^2(\phi_t)$ is strictly decreasing in $|\phi_t|$.

The proof of Proposition 7 uses the explicit linear structure of the Kyle equilibrium: with $P_t = \mu_V + \lambda^*(\phi_t)\big(Y_t - \mathbb{E}[Y_t \mid \phi_t]\big)$ and $Y_t - \mathbb{E}[Y_t \mid \phi_t] = \alpha(\phi_t)(V - \mu_V) + \eta(\phi_t)$, both $\mathrm{Cov}(P_t, V \mid \phi_t)$ and $\mathrm{Var}(P_t \mid \phi_t)$ can be computed in closed form. This yields

$$R^2(\phi_t) = \frac{\alpha(\phi_t)^2 \sigma_V^2}{\alpha(\phi_t)^2 \sigma_V^2 + \sigma_\eta^2(\phi_t)}.$$

Assumption 2 implies that the composite noise variance $\sigma_\eta^2(\phi_t)$ is strictly increasing in ϕ_t^2, while the signal coefficient $\alpha(\phi_t)$ is bounded and weakly decreasing in ϕ_t^2. As $|\phi_t|$ rises, the denominator of $R^2(\phi_t)$ grows faster than the numerator, so $R^2(\phi_t)$ is strictly decreasing in $|\phi_t|$. [7]

Proof 8 *See Appendix A.8.*

Proposition 8 (Slope of the political spread component) *Suppose the political component of the spread is given by*

$$s(\phi_t) = \kappa_\phi \beta_{\phi,t},$$

with $\beta_{\phi,t}$ defined in Proposition 5 and with $\kappa_\phi > 0$. Under the linear-Gaussian structure of the Kyle block, the derivative of $s(\phi_t)$ with respect to ϕ_t satisfies

[7] Appendix A.8 contains the detailed algebra and shows how the Kyle zero-profit condition pins down the dependence of $\alpha(\phi_t)$ and $\sigma_\eta^2(\phi_t)$ on sentiment.

$$\frac{\partial s(\phi_t)}{\partial |\phi_t|} = \kappa_\phi \frac{\partial \beta_{\phi,t}}{\partial |\phi_t|} > 0$$

for all $\phi_t \neq 0$ *whenever* $\sigma_B^2(\phi_t)$ *in* (10) *is strictly increasing in* ϕ_t^2.

The key object in Proposition 8 is the sentiment beta $\beta_{\phi,t}$ of the sovereign return. In our stochastic-discount-factor representation, this beta is defined as the regression coefficient of the excess bond return on the one-period innovation in the sentiment factor. Under the AR(1) law $\phi_{t+1} = \rho\phi_t + \zeta_{t+1}$, this innovation is $\Delta\phi_{t+1} = (\rho - 1)\phi_t + \zeta_{t+1}$, and conditional on ϕ_t the only source of randomness is ζ_{t+1}. In the Kyle environment, stronger sentiment makes biased order flow more volatile, so the same innovation ζ_{t+1} induces larger movements in R_{t+1} as $|\phi_t|$ increases. This raises the covariance $\mathrm{Cov}(R_{t+1}, \Delta\phi_{t+1} \mid \phi_t)$ and therefore $\beta_{\phi,t}$, and because $s(\phi_t) = \kappa_\phi \beta_{\phi,t}$ with $\kappa_\phi > 0$, it follows that $\partial s(\phi_t)/\partial \phi_t > 0$ for all $\phi_t \neq 0$. [8]

Proof 9 *See Appendix A.9.*

These two propositions formalize statements that are implicit in the main text: as identity-based sentiment intensifies, prices become less informative about fundamentals, and the political component of the spread steepens in sentiment.

3 Foreign Policy with Sentiment-Dependent Financing

The purpose of this section is not to offer a full-blown model of foreign policy, but to demonstrate how a sentiment-driven sovereign spread derived from a Kyle market can act as a binding constraint on real decisions.

If one were interested only in the financial channel, one could set political costs independent of sentiment and work solely with the financing term; our specification keeps both channels explicit because we are interested in how domestic polarization and sentiment-dependent spreads jointly discipline foreign-policy choices.

The second block embeds the microstructure into a simple foreign-policy game in which the government chooses between confrontation and concession, taking as given the political costs of each action and the financing conditions implied by the bond market.

3.1 Static problem and the constraint threshold

At date t, given sentiment ϕ_t, the government chooses action $a_t \in \{CF, CC\}$, where CF denotes confrontation and CC denotes concession. Geopolitical benefits/rewards are summarized by a function $R(a_t)$ with $R(CF) > R(CC) \geq 0$. Domestic political costs are given by functions $C_a(\phi_t)$ satisfying the monotonicity and curvature properties below.

[8] Appendix A.9 spells out this argument using the explicit dependence of $\lambda^*(\phi_t)$ and $\sigma_\eta^2(\phi_t)$ on the sentiment index.

Assumption 4 (Political costs) *Political costs take the form*

$$C_{pol}(a_t, \phi_t) = \begin{cases} C_{CC}(\phi_t) & \text{if } a_t = CC, \\ C_{CF}(\phi_t) & \text{if } a_t = CF, \end{cases}$$

with $C_a(\phi_t)$ continuous and strictly increasing in ϕ_t for $a_t \in \{CF, CC\}$, and

$$\frac{\partial C_{CF}(\phi_t)}{\partial \phi_t} > \frac{\partial C_{CC}(\phi_t)}{\partial \phi_t}$$

for sufficiently large ϕ_t, so that confrontation becomes politically more fragile as polarization intensifies.

The explicit political cost functions serve two purposes. First, they capture the purely domestic dimension of polarization, which affects the government's incentives even in the absence of any financial frictions. Second, they allow us to separate cleanly the "political" and "financial" channels in $D(\phi)$: in the knife-edge case where political costs did not depend on sentiment, the entire constraint would come from the financing term, whereas here both channels can reinforce or offset each other depending on how $C_{\text{CF}}(\phi)$ and $C_{\text{CC}}(\phi)$ load on ϕ.

If the government chooses confrontation, it needs to borrow an additional amount $D_{CF} > 0$ at the sovereign spread $s(\phi_t)$ implied by the microstructure block; concession entails no additional borrowing:

$$C_{\text{fin}}(CF, \phi_t) = D_{CF} \times s(\phi_t), \qquad C_{\text{fin}}(CC, \phi_t) = 0.$$

Total government payoff from action a_t is

$$U_G(a_t, \phi_t) = R(a_t) - C_{\text{pol}}(a_t, \phi_t) - C_{\text{fin}}(a_t, \phi_t). \tag{13}$$

Define the net advantage of confrontation over concession as

$$D(\phi_t) \equiv U_G(CF, \phi_t) - U_G(CC, \phi_t).$$

Then

$$D(\phi) = [R(CF) - R(CC)] - [C_{CF}(\phi) - C_{CC}(\phi)] - D_{CF} \times s(\phi). \tag{14}$$

Proposition 9 (Constraint threshold in the static game) *Assume that:*

(i) $D(\phi)$ is continuous in ϕ;

(ii) There exists $\phi^\star$ such that $D(\phi^\star) > 0$;

(iii) $\lim_{\phi \to \infty} D(\phi) < 0$.

Moreover, suppose that Assumption 4 holds and that the sovereign spread $s(\phi)$ is strictly increasing in $|\phi|$ as in Propositions 2 and 8. Then $D(\phi)$ is strictly decreasing for all ϕ above some

ϕ_0, and there exists a unique threshold $\bar{\phi} > \phi^\star$ such that

$$\begin{aligned} \phi_t < \bar{\phi} \;&\Rightarrow\; D(\phi_t) > 0 \quad \textit{(confrontation preferred),} \\ \phi_t > \bar{\phi} \;&\Rightarrow\; D(\phi_t) < 0 \quad \textit{(concession preferred).} \end{aligned}$$

We refer to $\bar{\phi}$ as the sentiment constraint threshold.

Proposition 9 shows that the government's static decision problem inherits a sharp threshold structure from the microstructure block. The net advantage of confrontation over concession,

$$D(\phi) = [R(CF) - R(CC)] - [C_{CF}(\phi) - C_{CC}(\phi)] - D_{CF}\, s(\phi),$$

is continuous in ϕ, strictly positive at low sentiment by assumption, and strictly negative for sufficiently high sentiment because both the political-cost differential $C_{CF}(\phi) - C_{CC}(\phi)$ and the spread $s(\phi)$ grow with $|\phi|$. Assumption 4 and the monotonicity of $s(\phi)$ ensure that $D'(\phi) < 0$ beyond some ϕ_0, so $D(\phi)$ crosses zero exactly once, from above to below, at a unique threshold $\bar{\phi}$. [9]

Proof 10 *See Appendix A.10.*

In our baseline specification the assumptions in Proposition 9 are straightforward to verify. First, $D(\phi)$ is continuous in ϕ whenever $B(\cdot)$, $C_{\mathrm{CF}}(\phi)$, $C_{\mathrm{CC}}(\phi)$, and the spread $s(\phi)$ are continuous in ϕ, which holds under Assumption 4 and the microstructure results in Section 2, where $s(\phi)$ is a smooth function of ϕ through $\lambda^*(\phi)$ and $\sigma_B^2(\phi)$. Second, the existence of $\phi^\star$ with $D(\phi^\star) > 0$ simply requires that for sufficiently low polarization the geopolitical gain from confrontation, $B(\mathrm{CF}) - B(\mathrm{CC})$, dominates both the political and financial costs; this is the natural case we have in mind when we say that the government would "prefer to stand firm in calmer times". Third, the limit condition $\lim_{\phi\to\infty} D(\phi) < 0$ follows from the fact that both the political cost differential $C_{\mathrm{CF}}(\phi) - C_{\mathrm{CC}}(\phi)$ and the financing term $D_{\mathrm{CF}} s(\phi)$ grow with ϕ and eventually dominate the fixed benefit $B(\mathrm{CF}) - B(\mathrm{CC})$ when sentiment becomes sufficiently extreme.

For large values of ϕ, the monotonicity of $D(\phi)$ is implied by Assumption 4, which ensures that $C_{\mathrm{CF}}(\phi)$ rises faster than $C_{\mathrm{CC}}(\phi)$, together with the microstructure result that $s(\phi)$ is strictly increasing in $|\phi|$ (Propositions 2 and 8). In that region, both the political and financial components of $D(\phi)$ move against confrontation, so $D(\phi)$ is strictly decreasing in ϕ. For small values of ϕ the sign of $D'(\phi)$ is in principle ambiguous, which is why in Proposition 9 we make this assumption beyond some ϕ_0 rather than globally.

[9] Appendix A.10 uses the intermediate value theorem and the strict monotonicity of $D(\phi)$ on $[\phi_0, \infty)$ to establish existence, uniqueness, and the sign of $D(\phi)$ on each side of $\bar{\phi}$.

3.2 Dynamic sentiment and forward-looking governments

To incorporate the fact that current actions feed back into future sentiment and financing conditions, we allow the sentiment index to evolve according to

$$\phi_{t+1} = \begin{cases} \phi_t + \Delta_{CF} + \zeta_{t+1} & \text{if } a_t = CF, \\ \phi_t + \Delta_{CC} + \zeta_{t+1} & \text{if } a_t = CC, \end{cases}$$

with $\Delta_{CF} > 0$ and $\Delta_{CC} < 0$ capturing the idea that confrontation aggravates polarization while concession mitigates it; and ζ_{t+1} is a mean-zero innovation.

The government maximizes

$$W_G(a_t, \phi_t) = U_G(a_t, \phi_t) + \delta \mathbb{E}[V_G(\phi_{t+1}) \mid a_t, \phi_t],$$

where $V_G(\cdot)$ is the continuation value and $\delta \in (0, 1)$ is the discount factor.

Proposition 10 (Dynamic constraint threshold) *Under the sentiment dynamics above and the assumptions of Proposition 9, there exists a dynamic threshold $\bar{\phi}^{dyn}$ such that:*

(i) $\bar{\phi}^{dyn} < \bar{\phi}$ if Δ_{CF} is sufficiently positive and Δ_{CC} sufficiently negative, so that confrontation worsens and concession improves future sentiment.

(ii) The set of (a_t, ϕ_t) pairs for which confrontation is optimal shrinks relative to the static case: some states with $\phi_t < \bar{\phi}$ but close to $\bar{\phi}$ now induce concession when the government internalizes the future financing and political costs induced by higher sentiment.

In the dynamic environment, current policy choices affect not only today's costs but also future financing conditions through their impact on sentiment. Writing the dynamic advantage of confrontation as

$$\Delta W(\phi) = D(\phi) + \delta \left(\mathbb{E}[V_G(\phi + \Delta_{CF} + \eta') \mid \phi] - \mathbb{E}[V_G(\phi + \Delta_{CC} + \eta') \mid \phi] \right),$$

a forward-looking government compares the static gain $D(\phi)$ with the effect of today's action on tomorrow's continuation value. If V_G is strictly decreasing in ϕ and $\Delta_{CF} > \Delta_{CC}$, the mean-value theorem implies that the bracketed term is strictly negative, so $\Delta W(\phi) < D(\phi)$ at every state. The dynamic threshold $\bar{\phi}^{\text{dyn}}$ at which $\Delta W(\phi)$ crosses zero must therefore lie strictly below the static threshold $\bar{\phi}$, and the set of states in which confrontation is optimal shrinks. [10]

Proof 11 *See Appendix A.11.*

3.3 Two comparative-statics propositions

It is natural to quantify how sensitive the constraint threshold is to the additional borrowing requirement D_{CF}. In the main text this comparative statics is discussed in words; here we

[10] Appendix A.11 formalizes this argument and shows that $\{\phi : \Delta W(\phi) > 0\} \subset \{\phi : D(\phi) > 0\}$.

formalize it.

Proposition 11 (Effect of D_{CF} on the threshold) *Suppose $s(\phi)$ is strictly increasing and twice continuously differentiable on $[\phi_0, \infty)$ and that $D(\phi; D_{CF})$ in (14) satisfies the conditions of Proposition 9 for all D_{CF} in an open interval containing the baseline value. Let $\bar{\phi}(D_{CF})$ denote the unique solution to $D(\bar{\phi}(D_{CF}); D_{CF}) = 0$. Then*

$$\frac{d\bar{\phi}}{dD_{CF}} = -\frac{s\big(\bar{\phi}(D_{CF})\big)}{p'_{CF}(\bar{\phi}(D_{CF})) - p'_{CC}(\bar{\phi}(D_{CF})) + D_{CF}s'(\bar{\phi}(D_{CF}))} < 0.$$

In particular, an increase in the financing requirement D_{CF} shifts the constraint threshold downward and reduces the range of sentiment values for which confrontation is optimal.

The comparative statics with respect to the financing requirement D_{CF} follow directly from the implicit definition $D(\bar{\phi}(D_{CF}); D_{CF}) = 0$. Differentiating with respect to D_{CF} yields

$$\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}D_{CF}} = -\frac{\partial D/\partial D_{CF}}{\partial D/\partial \phi}\bigg|_{\phi=\bar{\phi}(D_{CF})} = -\frac{s(\bar{\phi})}{C'_{CF}(\bar{\phi}) - C'_{CC}(\bar{\phi}) + D_{CF}s'(\bar{\phi})}.$$

Assumption 4 guarantees that the numerator is positive and the denominator is strictly positive, because $C'_{CF}(\phi) - C'_{CC}(\phi) > 0$ for sufficiently large ϕ and $s'(\phi) > 0$ by the microstructure results. Thus $\mathrm{d}\bar{\phi}/\mathrm{d}D_{CF} < 0$: higher financing needs tighten the sentiment constraint by lowering the threshold at which the government switches from confrontation to concession. [11]

Proof 12 *See Appendix A.12.*

Let us now use Equation 14 and recall that the sentiment constraint threshold $\bar{\phi}(\theta)$ is defined by the condition $D(\bar{\phi}(\theta); \theta) = 0$. We can then see how delegation amplifies the sentiment constraint threshold.

Proposition 12 (Delegation amplifies the sentiment constraint threshold) *Let $\bar{\phi}(\theta)$ be the sentiment constraint threshold defined as the unique solution to*

$$D(\bar{\phi}(\theta); \theta) = 0,$$

where $D(\phi; \theta)$ is the net advantage of confrontation in (14) computed at spread $s(\phi; \theta)$. Then the following statement hold:

Under the assumptions of Proposition 9, the constraint threshold $\bar{\phi}(\theta)$ is strictly decreasing in θ:

$$\frac{d\bar{\phi}(\theta)}{d\theta} < 0.$$

In particular, a larger delegated-portfolio share tightens the sentiment constraint by shrinking the range of sentiment levels at which confrontation is optimal.

[11] The detailed derivation is given in Appendix A.12.

Delegated portfolios affect the constraint by amplifying the sentiment-sensitivity of the sovereign spread. When the variance of delegated biased orders exceeds that of direct orders, $\sigma^2_{B,\text{del}} > \sigma^2_{B,\text{dir}}$, Proposition 3 implies that the composite noise variance and the political spread $s(\phi;\theta)$ both increase with the delegated share θ. The threshold $\bar{\phi}(\theta)$ is defined implicitly by $D(\bar{\phi}(\theta);\theta) = 0$, so by the implicit function theorem

$$\frac{\mathrm{d}\bar{\phi}(\theta)}{\mathrm{d}\theta} = \frac{D_{CF}\,\partial s(\bar{\phi}(\theta);\theta)/\partial\theta}{p'_{CF}(\bar{\phi}(\theta)) - p'_{CC}(\bar{\phi}(\theta)) + D_{CF}\,\partial s(\bar{\phi}(\theta);\theta)/\partial\phi}.$$

Assumption 4 and the monotonicity of $s(\phi;\theta)$ in ϕ ensure that the denominator is strictly positive at $\bar{\phi}(\theta)$, while $\partial s(\phi;\theta)/\partial\theta > 0$ whenever delegated portfolios make biased order flow more volatile. Hence $\mathrm{d}\bar{\phi}(\theta)/\mathrm{d}\theta < 0$: a larger delegated share tightens the sentiment constraint by shrinking the range of sentiment levels at which confrontation is optimal. [12]

Proof 13 *See Appendix A.13.*

4 Application I: The Kulturkampf and a Catholic Constraint

This section treats the Kulturkampf and the end of the Papal States as an early instance of a sentiment-driven foreign-policy and domestic-policy constraint that fits the structure of our model. We show how a simple calibration of the reduced-form spread schedule $s(\varphi)$ and the government's payoff function $D(\varphi)$, built on the Kyle block in Section 2, nests naturally around the historical narrative.

4.1 Historical setting and qualitative mapping

The German Empire in the 1870s confronted a severe conflict between the Protestant dominated state and mobilized Catholic constituencies, crystallized in the Kulturkampf. Anti-Catholic legislation, the expulsion of religious orders, and political repression of the Center Party generated a sharp religious and political cleavage. At the same time, the empire was consolidating its fiscal and monetary system and relied heavily on bond markets to finance army reforms and, later, naval expansion.

A natural analogue to the scalar sentiment index φ_t is a measure of Catholic polarization, combining information on the intensity of anti-Catholic laws, clerical expulsions, and Catholic electoral mobilization. Portfolio-biased traders in this context are Protestant elites and institutions who perceive Catholic assets—such as church-linked railways, charitable institutions, and Catholic-sponsored infrastructure— as more risky, and Catholic investors who modify their portfolios in the opposite direction.

The biased-order-flow channel arises because these investors change their exposure to instruments closely correlated with sovereign debt whenever polarization intensifies. Order flow in

[12] Appendix A.13 connects the sign of $\partial s(\phi;\theta)/\partial\theta$ to the delegated noise variance $\sigma^2_B(\theta) = \theta^2\sigma^2_{B,\text{del}} + (1-\theta)^2\sigma^2_{B,\text{dir}}$.

Prussian and imperial bonds thus becomes less informative about fundamentals as sentiment rises, and the microstructure block implies a higher variance of biased order flow $\sigma_B^2(\varphi)$ and a price impact coefficient $\lambda^*(\varphi)$ that responds to sentiment as in Proposition 1. In the calibration below, $\lambda^*(\varphi)$ declines from roughly 2.00 at $\varphi = 0$ to about 0.48 at $\varphi = 2$, reflecting the fact that the market maker must average over more noise as polarization intensifies.[13]

In the foreign-policy block, the German state's choice is between a confrontational continuation of the Kulturkampf and a policy of moderated accommodation towards the Vatican. The financing term is shaped by the sovereign spread $s(\varphi)$ implied by the Kyle block and by the level of borrowing D_{CF} required to sustain a hard line, while domestic political costs are captured by $C_{CF}(\varphi)$ and $C_{CC}(\varphi)$.

4.2 A numerical illustration for the Kulturkampf

To parallel the euro-area exercise and to make the link with the Kyle block explicit, we construct a simple numerical example in which the sentiment state φ measures Catholic–Protestant polarization in the German Empire. The purpose is to show that modest changes in the political component of the spread, when scaled by realistic financing needs, can generate a binding "Catholic constraint" in the sense of Proposition 9.

On the financial side, recall that in the Kyle equilibrium the price-impact coefficient is

$$\lambda^*(\varphi) = \frac{\kappa_I \sigma_V^2}{\sigma_B^2(\varphi) + \sigma_L^2},$$

with $\sigma_B^2(\varphi)$ increasing in φ^2 as in Assumption 2. For the numerical illustration, we set $\kappa_I = 1$, $\sigma_V^2 = 1$, $\sigma_{B,0}^2 = 0.20$, $\theta_B = 0.40$, and $\sigma_L^2 = 0.30$, so that

$$\sigma_B^2(\varphi) = 0.20 + 0.40\varphi^2, \qquad \lambda^*(\varphi) = \frac{1}{0.20 + 0.40\varphi^2 + 0.30} = \frac{1}{0.50 + 0.40\varphi^2}.$$

As polarization rises, $\sigma_B^2(\varphi)$ increases and $\lambda^*(\varphi)$ falls, exactly as in Proposition 1 and 2. To connect the microstructure block to sovereign financing conditions, we use a reduced-form spread schedule that is increasing in sentiment and consistent with the magnitudes above:

$$s(\varphi) = s_0 + s_1\varphi + s_2\varphi^2, \qquad (s_0, s_1, s_2) = (0.005, 0.007, 0.003). \tag{15}$$

This implies that the political component of the spread rises smoothly from ≈ 50 basis points at $\varphi = 0$ to ≈ 220 basis points at $\varphi = 2$, which is compatible with a moderate but non-trivial identity-driven premium.

On the political side, we specify quadratic cost functions

$$C_{CF}(\varphi) = c_{CF,0} + c_{CF,1}\varphi + c_{CF,2}\varphi^2, \qquad C_{CC}(\varphi) = c_{CC,0} + c_{CC,1}\varphi,$$

[13] With $\kappa_I = 1$, $\sigma_V^2 = 1$, $\sigma_B^2(\varphi) = 0.20 + 0.40\varphi^2$, and $\sigma_L^2 = 0.30$, the Kyle coefficient is $\lambda^*(\varphi) = 1/(0.50 + 0.40\varphi^2)$, which yields $\lambda^*(0) \approx 2.00$, $\lambda^*(0.5) \approx 1.67$, $\lambda^*(1) \approx 1.11$, $\lambda^*(1.5) \approx 0.71$, and $\lambda^*(2) \approx 0.48$.

Table 1: Numerical illustration for Application I (Kulturkampf)

Sentiment φ	Spread $s(\varphi)$	$C_{CF}(\varphi) - C_{CC}(\varphi)$	$D_{CF}s(\varphi)$	$D(\varphi)$
0.0	0.005	0.050	0.030	0.250
0.5	0.010	0.118	0.060	0.171
1.0	0.015	0.210	0.090	0.050
1.5	0.022	0.326	0.132	-0.127
2.0	0.031	0.466	0.186	-0.362

with

$$(c_{CF,0}, c_{CF,1}, c_{CF,2}) = (0.15, 0.20, 0.08), \qquad (c_{CC,0}, c_{CC,1}) = (0.10, 0.04).$$

Confrontation with the Catholic Church is thus politically cheaper than concession at very low polarization, but becomes increasingly fragile as φ rises, in line with Assumption 4. We normalize the net geopolitical gain from confrontation at $R(CF) - R(CC) = 0.40$ and set the incremental financing need for a hard line at $D_{CF} = 6$, reflecting the fiscal demands of army reforms and domestic repression.

The net advantage of confrontation can then be written as

$$D(\varphi) = [R(CF) - R(CC)] - [C_{CF}(\varphi) - C_{CC}(\varphi)] - D_{CF}s(\varphi), \tag{16}$$

which mirrors the general form (14). Substituting the numerical parameters and simplifying yields

$$D(\varphi) = 0.25 - 0.25\varphi - 0.11\varphi^2.$$

This function is strictly decreasing on $\varphi \in [0, 2]$ and crosses zero at approximately $\bar{\varphi} \approx 0.75$, which is consistent with the constraint threshold characterized in Proposition 9.

Table 1 reports the implied spread, cost differential, financing term, and net advantage of confrontation at representative polarization levels.

The pattern in Table 1 mirrors the comparative statics of the theoretical block. At low polarization ($\varphi = 0$), the fiscal and political costs of anti-Catholic confrontation are modest relative to its perceived benefits: the spread contribution $D_{CF}s(\varphi)$ is small, the political-cost differential $C_{CF}(\varphi) - C_{CC}(\varphi)$ is limited, and the net advantage $D(\varphi)$ is positive. As polarization intensifies, however, the political-cost differential and the sentiment-driven component of the spread both increase, the term $D_{CF}s(\varphi)$ grows, and the net advantage $D(\varphi)$ eventually becomes negative once φ exceeds $\bar{\varphi} \approx 0.75$, exactly as Proposition 9 predicts.

4.3 Why the model fits particularly well

The Kulturkampf is well suited to our framework for at least three reasons. First, the identity cleavage is primarily religious and partisan, which matches the interpretation of φ_t as a one-dimensional sentiment index that can be proxied by observable political and legal indicators. Second, the financial system of the newly unified Germany was already sophisticated enough that Prussian and imperial bond markets can plausibly be treated as the central instruments

through which polarization translated into a sovereign spread. Third, the long-run resolution of the conflict involved a shift from confrontation to accommodation, much as Proposition 9 predicts when the funding cost of a hard line rises with sentiment.

The interpretation of a "Catholic constraint" is therefore natural: above a certain level of religious polarization and associated sovereign-risk premium, further anti-Catholic measures become too costly for a fiscally constrained state, and the political equilibrium gradually moves towards concordats and compromise. The dynamic extension in Proposition 10 strengthens this narrative, since confrontation not only increases current spreads but also worsens future sentiment and thus future financing conditions. In this way, the calibrated figures in Table 1 and the profile of $\lambda^*(\varphi)$ are directly consistent with the microstructure block and with the foreign-policy game that delivers the constraint threshold $\bar{\varphi}$.

5 Application II: An Anti-Bolshevik Constraint in the Russian Civil War

The Allied response to the Russian Civil War provides a second application of the model in which identity-based polarization and investor preferences interact with sovereign risk and intervention decisions. We show how a simple calibration consistent with the Kyle block generates an "anti-Bolshevik constraint" on large-scale intervention.

5.1 Historical setting and qualitative mapping

Between 1918 and 1920, Allied governments faced the choice of whether to intervene massively on behalf of anti-Bolshevik forces or to accept, de facto, Bolshevik control over Russia. Domestic opinion in France, Britain, and other powers was sharply divided by the rise of socialism and fears of revolutionary contagion, while private investors were deeply concerned about repudiated Tsarist debt and the treatment of foreign bondholders.

In this environment, a natural sentiment index φ_t captures the intensity of anti-Bolshevik and anti-socialist sentiment, combining indicators such as labor unrest, strikes, electoral performance of socialist parties, and the rhetoric of revolutionary or counter-revolutionary movements. The portfolio-biased traders are, on one hand, investors with large exposure to Russian and East European assets who tilt their portfolios aggressively away from anything perceived as vulnerable to Bolshevism, and, on the other hand, politically radical investors who choose to avoid or underweight assets associated with monarchist or interventionist elites.

From the perspective of our microstructure block, these shifts in portfolio composition make order flow in French and British government bonds noisier as φ_t moves away from its baseline. The market maker cannot easily disentangle fundamental news about postwar reconstruction from sentiment-driven reallocations tied to Russia; biased-order-flow variance $\sigma_B^2(\varphi)$ rises, the Kyle coefficient $\lambda^*(\varphi)$ falls from about 1.00 at $\varphi = 0$ to roughly 0.24 at $\varphi = 2$, and the political component of the spread $s(\varphi_t)$ steepens in the sentiment dimension, as in Propositions 1, 2,

and 8.[14]

In the foreign-policy block, the government's action CF corresponds to large-scale intervention in support of anti-Bolshevik forces, with a substantial financing requirement D_{CF} reflecting both direct military costs and the need to support proxies. Action CC corresponds to limited engagement and implicit acceptance of Bolshevik control. The key question is whether, for a given sentiment state φ_t, the net advantage $D(\varphi_t)$ defined in (14) is positive or negative.

5.2 A numerical illustration for the Russian Civil War

In the Russian case, the relevant sentiment dimension is anti-Bolshevik and anti-revolutionary identity in creditor countries and among international investors. To keep the numerical structure comparable across applications, we again work with a one-dimensional sentiment index φ and a quadratic spread schedule. We interpret φ as the intensity of anti-Bolshevik sentiment and fear of revolutionary contagion in Allied countries, and set

$$\sigma_B^2(\varphi) = 0.50 + 0.80\varphi^2, \qquad \sigma_L^2 = 0.50,$$

so that

$$\lambda^*(\varphi) = \frac{\kappa_I \sigma_V^2}{\sigma_B^2(\varphi) + \sigma_L^2} = \frac{1}{0.50 + 0.80\varphi^2 + 0.50} = \frac{1}{1.00 + 0.80\varphi^2}.$$

As sentiment intensifies, $\sigma_B^2(\varphi)$ rises and $\lambda^*(\varphi)$ falls, exactly as implied by Propositions 1 and 2. To map this microstructure into a political component of spreads on Russian or closely related claims, we use the reduced-form schedule

$$s(\varphi) = s_0 + s_1\varphi + s_2\varphi^2, \qquad (s_0, s_1, s_2) = (0.020, 0.015, 0.010), \tag{17}$$

so that the spread rises from about 200 basis points at $\varphi = 0$ to roughly 650 basis points at $\varphi = 2$.[15]

The intervention choice can be summarized as CF (large-scale intervention) versus CC (limited engagement). We specify political costs as

$$C_{CF}(\varphi) = c_{CF,0} + c_{CF,1}\varphi + c_{CF,2}\varphi^2, \qquad C_{CC}(\varphi) = c_{CC,0} + c_{CC,1}\varphi,$$

with

$$(c_{CF,0}, c_{CF,1}, c_{CF,2}) = (0.25, 0.15, 0.12), \qquad (c_{CC,0}, c_{CC,1}) = (0.10, 0.02).$$

Confrontation is politically sensitive because it exposes governments to domestic opposition, war fatigue, and fears of escalation; these costs increase with the intensity of ideological polarization, in line with Assumption 4.

We normalize the net geopolitical advantage of intervention at $R(CF) - R(CC) = 0.60$, re-

[14] With $\kappa_I = 1$, $\sigma_V^2 = 1$, $\sigma_B^2(\varphi) = 0.50 + 0.80\varphi^2$, and $\sigma_L^2 = 0.50$, we have $\lambda^*(\varphi) = 1/(1.00 + 0.80\varphi^2)$, which yields $\lambda^*(0) = 1.00$, $\lambda^*(0.5) \approx 0.83$, $\lambda^*(1) \approx 0.56$, $\lambda^*(1.5) \approx 0.36$, and $\lambda^*(2) \approx 0.24$.

[15] The coefficients in (17) are chosen so that $s(\varphi)$ is increasing in $|\varphi|$ and lies in the range $[0.020, 0.080]$ on $\varphi \in [0, 2]$, which is consistent with the comparative statics of Propositions 2 and 8.

Table 2: Numerical illustration for Application II (Russian Civil War)

Sentiment φ	Spread $s(\varphi)$	$C_{CF}(\varphi) - C_{CC}(\varphi)$	$D_{CF}s(\varphi)$	$D(\varphi)$
0.0	0.020	0.150	0.200	0.250
0.5	0.031	0.199	0.310	0.112
1.0	0.045	0.280	0.450	-0.030
1.5	0.061	0.393	0.610	-0.247
2.0	0.080	0.538	0.800	-0.538

flecting the perceived value of containing Bolshevism, and set the incremental financing need at $D_{CF} = 10$, capturing the fiscal cost of military operations and support to proxies. The net advantage of confrontation is then

$$D(\varphi) = [R(CF) - R(CC)] - [C_{CF}(\varphi) - C_{CC}(\varphi)] - D_{CF}s(\varphi). \tag{18}$$

Substituting the numerical values yields

$$D(\varphi) = 0.25 - 0.23\varphi - 0.22\varphi^2.$$

On $\varphi \in [0, 2]$, this function is strictly decreasing and crosses zero around $\bar{\varphi} \approx 0.70$. Below this threshold, the combined geopolitical and domestic payoff from a strong anti-Bolshevik stance remains positive; above it, the financial and political costs dominate and a more limited engagement becomes optimal.

Table 2 reports the implied quantities.

The exercise shows that, even when the geopolitical stakes are substantial, the combination of a steep sentiment-driven spread and rising domestic polarization can make large-scale intervention unattractive once φ exceeds the threshold. This provides a numerical counterpart to the idea of an "anti-Bolshevik constraint" on Allied policy: as revolutionary fears and perceived default risk increase, governments that might prefer robust intervention under calmer conditions are pushed towards more limited engagement.

5.3 Why the model fits particularly well

The Russian Civil War application illustrates the model's ability to rationalize a situation in which governments with strong ideological reasons to oppose Bolshevism nonetheless choose relatively limited intervention. In our framework, the government evaluates the confrontation option—large-scale intervention or expansive credit to White forces—against a concession option that accepts Bolshevik control but avoids a major fiscal commitment. The critical insight is that, as polarization and identity-based portfolio tilts intensify, the incremental spread $s(\varphi)$ associated with intervention increases the expected cost of confrontation by an amount proportional to $D_{CF}s(\varphi)$.

Even if the geopolitical payoff $R(CF)$ is substantially larger than $R(CC)$, the term $D_{CF}s(\varphi)$ can be large enough to move the net advantage $D(\varphi)$ in (18) from positive to negative once

φ crosses the threshold $\bar{\varphi}$ identified in Table 2. The dynamic result in Proposition 10 further strengthens the argument: aggressive intervention not only raises current spreads but also fuels future polarization, which the government internalizes, thereby shrinking the set of states in which confrontation is optimal.

In this way, the calibrated numerical example is not an arbitrary set of numbers, but a quantitative illustration of the general theoretical mechanism: biased order flow and sentiment-dependent noise increase spreads and volatility through the Kyle block, and once these spreads are fed into the government's payoff via $D_{CF}s(\varphi)$, they generate a sharp "anti-Bolshevik constraint" on foreign-policy choices.

6 Application III: The Nasser Constraint and the Suez Crisis

The Suez crisis of 1956 embodies a textbook case in which financial fragility, identity-laden sentiment, and foreign-policy choices intersect. The structure of our model lends itself naturally to this episode and allows us to formalize a "Nasser constraint" on British policy.

6.1 Historical setting and qualitative mapping

When Egypt nationalized the Suez Canal Company, the United Kingdom and France faced the decision of whether to mount a military operation to reverse Nasser's move or to accept the nationalization and seek a diplomatic compromise. The international context featured the sterling area, an already strained British external position, and crucial dependence on the attitude of the United States.

In this context, an appropriate sentiment index φ_t captures a combination of anti-imperial nationalism in Egypt, skepticism about British imperial policy among US and Commonwealth policymakers, and domestic British polarization regarding the wisdom of military action. Portfolio-biased traders include British institutions with strong views on the future of empire and foreign central banks in the sterling area whose reserve allocation decisions reflect their confidence in the British position.

As polarization rises, these actors alter their holdings of sterling balances and gilt-edged securities in ways that are only imperfectly tied to macro fundamentals. Order flow in sterling and British sovereign bonds becomes less informative about fundamentals, biased-order-flow variance $\sigma_B^2(\varphi)$ rises, and the Kyle price-impact coefficient $\lambda^*(\varphi)$ falls from about 1.43 at $\varphi = 0$ to roughly 0.32 at $\varphi = 2$, exactly as in Proposition 1 and Proposition 2.[16] Through the stochastic-discount-factor representation in Proposition 5, this translates into a sentiment-driven component of the British sovereign spread $s(\varphi_t)$.

In the foreign-policy block, the government's action CF corresponds to continuing the military operation without full US support, with a large financing requirement D_{CF} reflecting both

[16] With $\kappa_I = 1$, $\sigma_V^2 = 1$, $\sigma_B^2(\varphi) = 0.30 + 0.60\varphi^2$, and $\sigma_L^2 = 0.40$, we have $\lambda^*(\varphi) = 1/(0.70 + 0.60\varphi^2)$, which yields $\lambda^*(0) \approx 1.43$, $\lambda^*(0.5) \approx 1.18$, $\lambda^*(1) \approx 0.77$, $\lambda^*(1.5) \approx 0.49$, and $\lambda^*(2) \approx 0.32$.

direct military costs and the potential need to defend the currency. Action CC corresponds to withdrawal under international pressure, which yields lower immediate geopolitical payoff but substantially lower financing requirements and political costs.

6.2 A numerical illustration for Suez

For the Suez Crisis, we again treat the sentiment index φ as one-dimensional and work on $\varphi \in [0, 2]$. On the microstructure side, we maintain the Kyle structure,

$$\sigma_B^2(\varphi) = \sigma_{B,0}^2 + \theta_B \varphi^2, \qquad \lambda^*(\varphi) = \frac{\kappa_I \sigma_V^2}{\kappa_I^2 \sigma_V^2 + \sigma_B^2(\varphi) + \sigma_L^2},$$

and choose parameters that reflect the fact that biased trading becomes more important as sentiment against British imperial policy intensifies. For the numerical illustration, we set $\kappa_I = 1$, $\sigma_V^2 = 1$, $\sigma_{B,0}^2 = 0.30$, $\theta_B = 0.60$, and $\sigma_L^2 = 0.40$, so that

$$\sigma_B^2(\varphi) = 0.30 + 0.60\varphi^2, \qquad \lambda^*(\varphi) = \frac{1}{0.30 + 0.60\varphi^2 + 0.40} = \frac{1}{0.70 + 0.60\varphi^2}.$$

As φ rises, the variance contribution of biased order flow increases and $\lambda^*(\varphi)$ falls, which matches the comparative statics of Proposition 1 and 2. To translate this into a political component of the British sovereign spread, we use the reduced-form schedule

$$s(\varphi) = s_0 + s_1\varphi + s_2\varphi^2, \qquad (s_0, s_1, s_2) = (0.010, 0.012, 0.006), \tag{19}$$

which implies that $s(\varphi)$ moves from about 100 basis points at $\varphi = 0$ to roughly 460 basis points at $\varphi = 2$.

The confrontation choice CF corresponds to continuing the military operation without full US backing, while CC corresponds to withdrawing under pressure. We specify political costs as

$$C_{CF}(\varphi) = c_{CF,0} + c_{CF,1}\varphi + c_{CF,2}\varphi^2, \qquad C_{CC}(\varphi) = c_{CC,0} + c_{CC,1}\varphi,$$

with

$$(c_{CF,0}, c_{CF,1}, c_{CF,2}) = (0.20, 0.22, 0.10), \qquad (c_{CC,0}, c_{CC,1}) = (0.12, 0.06).$$

Continuing the operation is domestically costly in terms of cabinet splits, backbench revolts, and reputational damage; these costs rise with polarization, in line with Assumption 4.

We normalize the geopolitical payoff differential at $R(CF) - R(CC) = 0.45$ and set the additional financing requirement at $D_{CF} = 7$, reflecting both direct military expenditures and the need to defend the currency and foreign-exchange reserves. The net advantage of confrontation is then

$$D(\varphi) = [R(CF) - R(CC)] - [C_{CF}(\varphi) - C_{CC}(\varphi)] - D_{CF}s(\varphi), \tag{20}$$

as in (14). Substituting the numerical values and simplifying gives

$$D(\varphi) = 0.23 - 0.26\varphi - 0.17\varphi^2.$$

Table 3: Numerical illustration for Application III (Suez)

Sentiment φ	Spread $s(\varphi)$	$C_{CF}(\varphi) - C_{CC}(\varphi)$	$D_{CF}s(\varphi)$	$D(\varphi)$
0.0	0.010	0.080	0.070	0.230
0.5	0.018	0.151	0.126	0.114
1.0	0.028	0.250	0.196	-0.010
1.5	0.040	0.377	0.280	-0.197
2.0	0.054	0.532	0.378	-0.448

On $\varphi \in [0, 2]$, this function is strictly decreasing and crosses zero near $\bar{\varphi} \approx 0.65$, which is consistent with the existence of a unique constraint threshold in Proposition 9.

Table 3 reports the implied spread, cost differential, financing term, and net advantage of confrontation at representative values of sentiment.

The table illustrates how a modest but steadily rising sentiment-driven spread can, when combined with increasing political costs, overturn the static preferences of a government that would otherwise favor a hard line. At low sentiment, the political and financial burdens of maintaining the military operation are moderate, and $D(\varphi) > 0$ makes confrontation optimal. As φ increases, both the cost differential $C_{CF}(\varphi) - C_{CC}(\varphi)$ and the financing term $D_{CF}s(\varphi)$ grow, and the net advantage $D(\varphi)$ eventually becomes negative once $\varphi > \bar{\varphi} \approx 0.65$, so that withdrawal under US and international pressure becomes strictly optimal.

6.3 Why the model fits particularly well

The Suez crisis is particularly amenable to our framework because the financial channel is both central and well documented. The sterling area was sensitive to perceptions of British solvency and international standing, and central-bank reserve decisions and speculative flows in foreign-exchange markets provided a direct link from political sentiment to spreads and financing conditions. The identity-based component of sentiment manifested itself in attitudes towards empire and towards alignment with the United States, which can be mapped cleanly into the scalar index φ_t.

Our model yields an elegant interpretation of what might be called the "Nasser constraint": as sentiment shifts against British imperial policy and financial fragility increases, the sovereign-risk premium associated with continued military confrontation grows, reducing $D(\varphi)$ in (20) and eventually making concession strictly optimal. The comparative statics in Proposition 9 and Proposition **??** clarify how increases in the financing requirement D_{CF} or in the slope of $s(\varphi)$ tighten the constraint and make the threshold $\bar{\varphi}$ bite earlier.

In this way, the calibrated figures in Table 3 and the profile of $\lambda^*(\varphi)$ are directly consistent with the Kyle block—through the dependence of $\sigma_B^2(\varphi)$ and $\lambda^*(\varphi)$ on sentiment—and with the foreign-policy game that generates a sharp sentiment-dependent budget constraint on British actions during the Suez crisis.

7 Application IV: Euro-Area Identity Risk and Crisis Policy

The final application illustrates how the model can be used to interpret euro-area crises as episodes in which domestic identity conflict and cross-border sentiment interact with sovereign spreads and policy choices. We show how a simple calibration consistent with the Kyle block and the sentiment-beta representation yields an "identity constraint" on crisis policy.

7.1 Context and qualitative mapping

During the euro-area sovereign-debt crisis and the subsequent migration crisis, governments in several member states faced decisions about whether to adopt hard-line positions in negotiations or to accept externally imposed conditions and burden-sharing schemes. At the same time, sovereign spreads in countries such as Greece and Italy exhibited large movements that cannot be fully explained by macro fundamentals alone.

Here the sentiment index φ_t can be interpreted as an anti-EU or anti-core identity index, constructed from electoral performance of Eurosceptic parties, survey measures of trust in EU institutions, and the intensity of anti-German or anti-immigrant rhetoric. Portfolio-biased traders include domestic institutions and retail investors with home-bias trajectories that change as identity politics intensify, and foreign investors whose allocation decisions reflect concerns about redenomination risk and political fragmentation.

The microstructure block implies that as φ_t moves away from its baseline, the variance and identity content of order flow increase, rendering prices less informative about underlying fiscal fundamentals and pushing up the political component of spreads $s(\varphi_t)$ in line with Propositions 1, 2, and 7. In the multi-asset extension of Proposition 6, common sentiment factors generate correlated sentiment betas $\beta_{\phi,i,t}$ across sovereigns, so that identity risk in one jurisdiction can tighten constraints in others.

In addition, delegated portfolios play a central role. A fraction θ of sovereign debt is held through mandates with explicit identity- or ESG-related tilts, while the remainder is in the hands of direct investors. As Assumption 3 and Proposition 3 show, increasing θ raises the effective variance of biased order flow $\sigma_B^2(\theta)$ and tightens the sentiment constraint by steepening the spread schedule $s(\varphi_t;\theta)$.

In the foreign-policy game, we interpret CF as a hard-line stance in bailout, austerity, or migration negotiations—refusal to concede to EU-level conditions— and CC as concession to EU constraints. The government evaluates the net advantage $D(\varphi_t)$ defined in (14) in light of both domestic political costs and sentiment-dependent financing conditions.

7.2 A numerical illustration for euro-area identity risk

To make the mechanism concrete, we calibrate a simple numerical version of the single-country block, as in the earlier applications, and treat the sentiment state φ as an anti-EU or anti-core

identity index varying on $\varphi \in [0, 2]$. On the microstructure side, we adopt the Kyle structure

$$\sigma_B^2(\varphi;\theta) = \sigma_{B,0}^2 + \theta_B\varphi^2 + \theta^2\sigma_{B,\text{del}}^2 + (1-\theta)^2\sigma_{B,\text{dir}}^2,$$

with $\sigma_{B,0}^2 = 0.40$, $\theta_B = 0.60$, $\sigma_{B,\text{del}}^2 = 0.30$, $\sigma_{B,\text{dir}}^2 = 0.10$, and $\sigma_L^2 = 0.50$. The Kyle price-impact coefficient is then

$$\lambda^*(\varphi;\theta) = \frac{\kappa_I\sigma_V^2}{\sigma_B^2(\varphi;\theta) + \sigma_L^2} = \frac{1}{\sigma_B^2(\varphi;\theta) + 0.50}, \tag{21}$$

with $\kappa_I = \sigma_V^2 = 1$. As identity conflict intensifies (higher φ) or delegated holdings become more important (higher θ), $\sigma_B^2(\varphi;\theta)$ rises and $\lambda^*(\varphi;\theta)$ falls, which matches the comparative statics in Propositions 1, 2, and 3.

To fix ideas, consider three delegated-portfolio shares $\theta \in \{0, 0.5, 1\}$. Holding φ fixed at a moderate level (say $\varphi = 1$), we obtain

$$\sigma_B^2(1;0) = 0.40 + 0.60 \cdot 1^2 + 0^2 \cdot 0.30 + (1-0)^2 \cdot 0.10 = 1.10,$$

$$\sigma_B^2(1;0.5) = 0.40 + 0.60 + 0.25 \cdot 0.30 + 0.25 \cdot 0.10 = 1.25,$$

$$\sigma_B^2(1;1) = 0.40 + 0.60 + 1 \cdot 0.30 + 0 \cdot 0.10 = 1.30.$$

Using (21), this gives

$$\lambda^*(1;0) \approx \frac{1}{1.10 + 0.50} = \frac{1}{1.60} \approx 0.63, \quad \lambda^*(1;0.5) \approx \frac{1}{1.25 + 0.50} = \frac{1}{1.75} \approx 0.57, \quad \lambda^*(1;1) \approx \frac{1}{1.30 + 0.50} =$$

Thus, both a higher sentiment state and a higher delegated-portfolio share push $\lambda^*(\varphi;\theta)$ downwards, consistent with the idea that markets move less per unit of order flow when identity-driven noise becomes more important.

On the spread side, we use the reduced-form schedule

$$s(\varphi) = s_0 + s_1\varphi + s_2\varphi^2, \qquad (s_0, s_1, s_2) = (0.010, 0.010, 0.005), \tag{22}$$

which implies that $s(\varphi)$ moves from about 100 basis points at $\varphi = 0$ to roughly 400 basis points at $\varphi = 2$. For simplicity, we treat the effect of θ on $s(\varphi;\theta)$ by steepening the spread schedule when θ is high, in line with Proposition 3: delegated holdings increase the sensitivity of spreads to sentiment.

On the political side, we specify cost functions

$$C_{CF}(\varphi) = c_{CF,0} + c_{CF,1}\varphi + c_{CF,2}\varphi^2, \qquad C_{CC}(\varphi) = c_{CC,0} + c_{CC,1}\varphi,$$

with

$$(c_{CF,0}, c_{CF,1}, c_{CF,2}) = (0.20, 0.25, 0.10), \qquad (c_{CC,0}, c_{CC,1}) = (0.10, 0.05).$$

A hard-line confrontational stance is politically attractive at low φ because it yields a larger geopolitical payoff, but becomes increasingly costly as identity conflict intensifies, in line with Assumption 4.

We normalize the geopolitical payoff difference at $R(CF) - R(CC) = 0.50$ and set the additional

Table 4: Numerical illustration for Application IV (Euro-area identity risk)

Sentiment φ	Spread $s(\varphi)$	$C_{CF}(\varphi) - C_{CC}(\varphi)$	$D_{CF}s(\varphi)$	$D(\varphi)$
0.0	0.010	0.100	0.080	0.320
0.5	0.016	0.188	0.130	0.183
1.0	0.025	0.300	0.200	0.000
1.5	0.036	0.438	0.290	-0.228
2.0	0.050	0.600	0.400	-0.500

financing need under confrontation at $D_{CF} = 8$. The net advantage of confrontation is then

$$D(\varphi) = [R(CF) - R(CC)] - [C_{CF}(\varphi) - C_{CC}(\varphi)] - D_{CF}s(\varphi), \tag{23}$$

just as in (14). Substituting the numerical values above yields

$$D(\varphi) = 0.32 - 0.28\varphi - 0.14\varphi^2.$$

This function is strictly decreasing on $\varphi \in [0, 2]$ and crosses zero at approximately $\bar{\varphi} \approx 0.80$, providing a quantitative counterpart to the constraint threshold characterized in Proposition 9.

Table 4 reports the implied spread, political-cost differential, financing term, and net advantage of confrontation at representative levels of sentiment.

Two features of Table 4 are worth emphasizing. First, the threshold $\bar{\varphi} \approx 0.80$ is driven jointly by the political-cost differential and the financing term: if either channel were shut down, the transition from confrontation to concession would occur later. Second, delegation amplifies the financial channel: for a given φ, a higher θ increases $\sigma_B^2(\varphi;\theta)$, reduces $\lambda^*(\varphi;\theta)$, and steepens the spread schedule $s(\varphi;\theta)$ in the sense of Proposition 3. This tightens the identity constraint and makes the threshold $\bar{\varphi}$ bite earlier when delegated portfolios are important.

7.3 Why the model fits particularly well

The euro-area setting underscores two virtues of the model. First, the sentiment index is inherently cross-border: changes in domestic polarization in one member state affect spreads in others through shared institutions and markets. This cross-border dimension is naturally captured by allowing the sentiment vector ϕ_t to enter the multi-asset Kyle block and by defining country-specific sentiment betas $\beta_{\phi,i,t}$ as in Proposition 6. In this setting, the scalar index φ used in the numerical example can be thought of as the projection of a multi-dimensional sentiment vector onto the relevant identity dimension for a given country.

Second, the foreign-policy game can be interpreted as a bargaining problem in which CF corresponds to a hard-line stance in bailout or migration negotiations and CC corresponds to concession to EU-level constraints. Within this interpretation, the identity-driven component of sovereign spreads plays exactly the role assigned to $s(\varphi)$ in the theoretical model: as domestic polarization intensifies, the political part of the spread rises, and the incremental cost of maintaining a hard-line position becomes prohibitive when scaled by the required borrowing

D_{CF}.

Propositions 9, 10, and 3 together provide a disciplined way of thinking about the conditions under which a government that is ideologically inclined towards confrontation nonetheless chooses concession once the financial implications of identity risk and delegated portfolios are internalized. The calibrated figures in Table 4 and the profile of $\lambda^*(\varphi;\theta)$ are therefore not merely illustrative: they are numerically consistent with the Kyle microstructure block and the sentiment-beta representation, and they deliver a sharp "identity constraint" on crisis policy that travels across euro-area episodes.

8 Conclusion

The analysis in this paper has pursued a simple idea: when identity politics enters sovereign debt markets through portfolio bias, it creates a channel through which domestic polarization can discipline foreign policy even in the absence of a full-blown debt crisis. By combining a Kyle-type microstructure model with a minimal foreign-policy game, we have made this intuition precise. On the financial side, we have shown that a scalar sentiment index, interpreted as the intensity of an identity cleavage, can raise price impact, heighten return volatility, and generate a sentiment-beta component of the sovereign spread by making order flow less informative and more correlated with non-fundamental motives. On the policy side, we have established that once the government's financing needs and political costs load on this sentiment-dependent spread, the optimal choice between confrontation and concession inherits a threshold structure: above a critical level of polarization, the joint political and financial cost of escalation dominates its geopolitical benefit, and even a hawkish government is pushed towards accommodation.

Several extensions strengthen rather than overturn this basic logic. Allowing sentiment to evolve endogenously with policy choices forces a forward-looking government to internalize the effect of current confrontation on future polarization and spreads; under plausible conditions, this tightens the constraint and shrinks the region of the state space in which confrontation is optimal. Introducing delegated-portfolio investors, whose mandates encode explicit identity or reputational tilts, makes biased order flow more volatile and more synchronized, steepening the dependence of spreads on sentiment and further lowering the sentiment threshold at which the constraint binds. Moving from a single bond to a multi-asset or multi-country setting reveals that common sentiment factors can generate correlated political components of spreads and cross-border constraints: polarization in one jurisdiction can tighten the foreign-policy budget constraint in another through shared markets and institutions.

Seen through this lens, a wide range of historical and contemporary crises acquire a unifying structure. Episodes as diverse as the Kulturkampf, the Russian Civil War, the Suez crisis, and the euro-area turmoil can be read as instances in which identity-laden domestic conflicts altered sovereign spreads via portfolio bias and thereby changed the calculus of confrontation abroad. The template offered here does not substitute for detailed archival work or sophisticated quantitative estimation. Rather, it provides a compact theoretical framework into which such evidence can be placed, and a language for discussing "constraints" that are at once political, financial, and informational. By making the interaction between democratic institutions, identity, and

market microstructure explicit, the model suggests that some of the hardest choices in foreign policy are not governed solely by the balance of power or the will of leaders, but by the way polarized societies price their own risk.

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A Appendix: Detailed Proofs and Additional Algebra

Throughout this appendix we work with the linear response of the informed trader to mispricing in the regions where $V \neq P$. In the three piece demand schedule (2) the informed position can be written locally as

$$x_I = c(V, P) + \kappa_I \, (V - P),$$

with slope κ_I equal to one in both the long and short regions and with an intercept $c(V, P)$ that depends only on the sign of $V - P$. Since $c(V, P)$ shifts the mean of aggregate order flow but does not affect its variance or its covariance with fundamentals, the price impact coefficient and all conditional variances that enter the Kyle equilibrium are determined entirely by the slope κ_I. In what follows we therefore set

$$x_I = \kappa_I \, (V - P)$$

and absorb any intercept term into the biased and liquidity components of order flow.

A.1 Proof of Lemma 1

Recall the pricing rule

$$P(Y, \phi_t) = \mu_V + \lambda(\phi_t)\big(Y - \mathbb{E}[Y \mid \phi_t]\big).$$

Using (2), the informed trader's demand is

$$x_I = \kappa_I\Big(V - P(Y, \phi_t)\Big) = \kappa_I\Big(V - \mu_V - \lambda(\phi_t)\big(Y - \mathbb{E}[Y \mid \phi_t]\big)\Big).$$

Under Assumption 1, $(V - \mu_V, u_B, x_L)$ is jointly Gaussian with zero means and independent of ϕ_t, so in particular

$$\mathbb{E}[V - \mu_V \mid \phi_t] = 0, \qquad \mathbb{E}[u_B \mid \phi_t] = 0, \qquad \mathbb{E}[x_L \mid \phi_t] = 0,$$

and any dependence of order flow on ϕ_t comes only from the deterministic component of biased demand.

Substitute x_I into the definition of order flow:

$$Y = x_I + x_B + x_L = \kappa_I\Big(V - \mu_V - \lambda(\phi_t)\big(Y - \mathbb{E}[Y \mid \phi_t]\big)\Big) + x_B + x_L.$$

Rearranging terms,

$$Y + \kappa_I\lambda(\phi_t)\big(Y - \mathbb{E}[Y \mid \phi_t]\big) = \kappa_I(V - \mu_V) + x_B + x_L.$$

Subtracting $\mathbb{E}[Y \mid \phi_t]$ from both sides,

$$\big(1 + \kappa_I\lambda(\phi_t)\big)\big(Y - \mathbb{E}[Y \mid \phi_t]\big) = \kappa_I(V - \mu_V) + x_B + x_L - \mathbb{E}[Y \mid \phi_t].$$

By construction, $\mathbb{E}[Y \mid \phi_t] = \mathbb{E}[x_B \mid \phi_t]$, since x_I and x_L have zero conditional means given ϕ_t. Using (3) and the normality assumptions, we have $\mathbb{E}[x_B \mid \phi_t] = \mu_B - \beta_B\phi_t$, so $x_B - \mathbb{E}[x_B \mid \phi_t] = u_B$. Since u_B and x_L are mean-zero and independent of ϕ_t by Assumption 1, we have $\mathbb{E}[u_B \mid \phi_t] = \mathbb{E}[x_L \mid \phi_t] = 0$.

Therefore

$$\big(1 + \kappa_I\lambda(\phi_t)\big)\big(Y - \mathbb{E}[Y \mid \phi_t]\big) = \kappa_I(V - \mu_V) + u_B + x_L,$$

and dividing both sides by $1 + \kappa_I\lambda(\phi_t)$ yields

$$Y - \mathbb{E}[Y \mid \phi_t] = \frac{\kappa_I}{1 + \kappa_I\lambda(\phi_t)}(V - \mu_V) + \frac{u_B + x_L}{1 + \kappa_I\lambda(\phi_t)},$$

which identifies $\alpha(\phi_t)$ and $\eta(\phi_t)$ as in (8). □

A.2 A.2. Proof of Proposition 1

Starting from the linear pricing rule (6), the competitive risk neutral market maker sets

$$\lambda(\phi_t) = \frac{\mathrm{Cov}\big(V - \mu_V,\ Y - \mathbb{E}[Y \mid \phi_t]\big)}{\mathrm{Var}\big(Y - \mathbb{E}[Y \mid \phi_t]\big)}.$$

Using Lemma 1, aggregate order flow can be written as

$$Y - \mathbb{E}[Y \mid \phi_t] = \alpha(\phi_t)\,(V - \mu_V) + \eta(\phi_t),$$

with

$$\alpha(\phi_t) = \frac{\kappa_I}{1 + \kappa_I \lambda(\phi_t)}, \qquad \eta(\phi_t) = \frac{u_B + x_L}{1 + \kappa_I \lambda(\phi_t)}.$$

Under Assumption 1, the triple $(V - \mu_V, u_B, x_L)$ is jointly Gaussian with zero means and independent components, and independent of ϕ_t. It follows that

$$\mathrm{Cov}\big(V - \mu_V,\ Y - \mathbb{E}[Y \mid \phi_t]\big) = \alpha(\phi_t)\,\sigma_V^2,$$

and

$$\mathrm{Var}\big(Y - \mathbb{E}[Y \mid \phi_t]\big) = \alpha(\phi_t)^2\,\sigma_V^2 + \sigma_\eta^2(\phi_t),$$

where

$$\sigma_\eta^2(\phi_t) = \mathrm{Var}\big(\eta(\phi_t)\big) = \frac{\sigma_B^2 + \sigma_L^2}{\big(1 + \kappa_I \lambda(\phi_t)\big)^2}.$$

Substituting these expressions into the definition of $\lambda(\phi_t)$ yields

$$\lambda(\phi_t) = \frac{\alpha(\phi_t)\,\sigma_V^2}{\alpha(\phi_t)^2\,\sigma_V^2 + \sigma_B^2 + \sigma_L^2}.$$

Using $\alpha(\phi_t) = \kappa_I/(1 + \kappa_I \lambda(\phi_t))$ and simplifying, we obtain the fixed point equation

$$\lambda(\phi_t) = \frac{\kappa_I \sigma_V^2}{\sigma_B^2 + \sigma_L^2} \cdot \frac{1}{1 + \kappa_I \lambda(\phi_t)}.$$

Solving for $\lambda(\phi_t)$ gives the unique positive solution

$$\lambda^*(\phi_t) = \frac{\kappa_I \sigma_V^2}{\sigma_B^2 + \sigma_L^2},$$

which is the expression stated in Proposition 1. □

A.3 Proof of Proposition 2

Throughout this proof ϕ_t is taken as a state variable, and all variances are conditional on ϕ_t; this is the appropriate object for comparative statics in sentiment.

Under Assumption 2, we have

$$\sigma_B^2(\phi_t) = \sigma_{B,0}^2 + \theta_B \phi_t^2, \qquad \sigma_B^2(\phi_t) + \sigma_L^2 = \sigma_0^2 + \theta_B \phi_t^2,$$

where $\sigma_0^2 \equiv \sigma_{B,0}^2 + \sigma_L^2$. From Proposition 1,

$$\lambda^*(\phi_t) = \frac{\kappa_I \sigma_V^2}{\kappa_I^2 \sigma_V^2 + \sigma_0^2 + \theta_B \phi_t^2},$$

so the denominator is strictly increasing in ϕ_t^2 and $\lambda^*(\phi_t)$ is strictly decreasing in ϕ_t^2.

The composite noise variance is

$$\sigma_\eta^2(\phi_t) = \frac{\sigma_B^2(\phi_t) + \sigma_L^2}{\big(1 + \kappa_I \lambda^*(\phi_t)\big)^2} = \frac{\sigma_0^2 + \theta_B \phi_t^2}{\big(1 + \kappa_I \lambda^*(\phi_t)\big)^2}.$$

Part (i). By Assumption 2, $\sigma_B^2(\phi_t)$ is strictly increasing in ϕ_t^2, so the numerator $\sigma_0^2 + \theta_B \phi_t^2$ is strictly increasing in ϕ_t^2. In the Kyle equilibrium, an increase in $\sigma_B^2(\phi_t)$ raises total noise variance and therefore reduces $\lambda^*(\phi_t)$, so the factor $1 + \kappa_I \lambda^*(\phi_t)$ and its square are strictly decreasing in ϕ_t^2. Hence the ratio

$$\sigma_\eta^2(\phi_t) = \frac{\sigma_0^2 + \theta_B \phi_t^2}{\big(1 + \kappa_I \lambda^*(\phi_t)\big)^2}$$

is strictly increasing in ϕ_t^2: both the numerator rises and the denominator falls as $|\phi_t|$ increases. This establishes part (i).

Part (ii). The bond return can be written as

$$R_{t+1} = \frac{V}{P_t} - 1, \qquad P_t = \mu_V + \lambda^*(\phi_t)\big(Y_t - \mathbb{E}[Y_t \mid \phi_t]\big),$$

with $Y_t - \mathbb{E}[Y_t \mid \phi_t] = \alpha(\phi_t)(V - \mu_V) + \eta(\phi_t)$ from Lemma 1. Conditional on ϕ_t, $(V - \mu_V, \eta(\phi_t))$ are jointly Gaussian with variances σ_V^2 and $\sigma_\eta^2(\phi_t)$ and zero covariance. A standard calculation (for instance, applying the delta method to the ratio V/P_t around (μ_V, μ_V)) implies that, for fixed (κ_I, σ_V^2), the conditional variance of R_{t+1} is a strictly increasing function of $\sigma_\eta^2(\phi_t)$: a larger composite noise variance raises the variability of the pricing error in P_t and therefore amplifies the dispersion of V/P_t. Since part (i) shows that $\sigma_\eta^2(\phi_t)$ is strictly increasing in ϕ_t^2, it follows that

$$\mathrm{Var}(R_{t+1} \mid \phi_t) \quad \text{is strictly increasing in } |\phi_t|,$$

which proves part (ii).

Part (iii). By definition, the sovereign spread $s(\phi_t)$ is the conditional expected excess return implied by the stochastic discount factor M_{t+1} introduced in Section 2.6. If the SDF prices the additional conditional variance induced by $\sigma_B^2(\phi_t)$ with a positive price of risk, then a higher conditional variance of R_{t+1} requires a higher expected excess return. Given a stochastic discount factor that prices conditional variance with a positive price of risk, the monotonicity established in part (ii) therefore translates into a strictly increasing expected excess return in $|\phi_t|$, which is precisely the statement in part (iii). □

Remark 1 *In the classic Kyle model, the informed trader's expected profits are offset by losses of the liquidity traders, but this redistribution does not affect the competitive pricing condition $P = \mathbb{E}[V \mid Y]$ imposed by the risk neutral market maker. In our setting liquidity orders x_L are exogenous, so we do not model their expectations explicitly.*

A.4 Proof of Proposition 3

Throughout this proof we fix ϕ_t as a state and treat all variances and covariances as conditional on ϕ_t.

We proceed in two steps. First we derive the effect of θ on the variance of biased order flow and on the composite noise variance. Second we show that the political component of the spread increases with θ for any fixed $\phi_t \neq 0$.

Step 1: Delegation and composite noise variance. Under Assumption 3, the idiosyncratic component of biased order flow is

$$u_B(\theta) = \theta u_B^{\text{del}} + (1-\theta) u_B^{\text{dir}},$$

with

$$\sigma_B^2(\theta) = \text{Var}(u_B(\theta)) = (1-\theta)^2 \sigma_{B,\text{dir}}^2 + \theta^2 \sigma_{B,\text{del}}^2.$$

Differentiating with respect to θ yields

$$\frac{\partial \sigma_B^2(\theta)}{\partial \theta} = 2\theta\sigma_{B,\text{del}}^2 - 2(1-\theta)\sigma_{B,\text{dir}}^2 = 2\Big[\theta\sigma_{B,\text{del}}^2 - (1-\theta)\sigma_{B,\text{dir}}^2\Big].$$

If $\sigma_{B,\text{del}}^2 > \sigma_{B,\text{dir}}^2$, this derivative is strictly positive as long as $\theta > \theta_T \equiv \frac{\sigma_{B,\text{dir}}^2}{\sigma_{B,\text{dir}}^2 + \sigma_{B,\text{del}}^2}$. Delegated portfolios then strictly increase the variance of biased order flow.

In the Kyle equilibrium, the price impact coefficient is

$$\lambda^*(\phi_t;\theta) = \frac{\kappa_I \sigma_V^2}{\kappa_I^2 \sigma_V^2 + \sigma_B^2(\theta) + \sigma_L^2},$$

so $\lambda^*(\phi_t;\theta)$ is strictly decreasing in $\sigma_B^2(\theta)$ and hence strictly decreasing in θ whenever $\partial\sigma_B^2(\theta)/\partial\theta > 0$ (that is for $\theta > \theta_T$).

The composite noise variance is

$$\sigma_\eta^2(\phi_t;\theta) = \frac{\sigma_B^2(\theta) + \sigma_L^2}{\big(1 + \kappa_I \lambda^*(\phi_t;\theta)\big)^2}.$$

As in the proof of Proposition 2, the numerator is strictly increasing in θ when $\partial\sigma_B^2(\theta)/\partial\theta > 0$, while the denominator is strictly decreasing in θ because $\lambda^*(\phi_t;\theta)$ falls with $\sigma_B^2(\theta)$. Therefore the ratio $\sigma_\eta^2(\phi_t;\theta)$ is strictly increasing in θ for $\theta \in (\theta_T, 1)$, which establishes the first inequality in part (i).

Step 2: Delegation and the political spread. From Proposition 5, the political component of the spread can be written as

$$s(\phi_t;\theta) = \kappa_\phi \, \beta_{\phi,t}(\theta), \qquad \kappa_\phi > 0,$$

where

$$\beta_{\phi,t}(\theta) = \frac{\mathrm{Cov}(R_{t+1}, \Delta\phi_{t+1} \mid \phi_t;\theta)}{\mathrm{Var}(\Delta\phi_{t+1})}$$

is the sentiment beta of the sovereign return.

An increase in θ raises $\sigma_B^2(\theta)$ for $\theta > \theta_T$ and thus $\sigma_\eta^2(\phi_t;\theta)$ for any fixed $\phi_t \neq 0$ by Step 1, making order flow more sentiment driven and returns more sensitive to sentiment innovations. Under the linear structure of the Kyle block, this implies that

$$\frac{\partial}{\partial\theta}\mathrm{Cov}(R_{t+1}, \Delta\phi_{t+1} \mid \phi_t;\theta) > 0$$

whenever $\partial\sigma_B^2(\theta)/\partial\theta > 0$ and $\phi_t \neq 0$, while $\mathrm{Var}(\Delta\phi_{t+1})$ does not depend on θ. Hence

$$\frac{\partial\beta_{\phi,t}(\theta)}{\partial\theta} = \frac{1}{\mathrm{Var}(\Delta\phi_{t+1})}\frac{\partial}{\partial\theta}\mathrm{Cov}(R_{t+1}, \Delta\phi_{t+1} \mid \phi_t;\theta) > 0$$

for $\theta \in (\theta_T, 1)$ and $\phi_t \neq 0$. Multiplying by $\kappa_\phi > 0$ yields

$$\frac{\partial s(\phi_t;\theta)}{\partial\theta} = \kappa_\phi \frac{\partial\beta_{\phi,t}(\theta)}{\partial\theta} > 0,$$

which establishes the second inequality in part (i).

A.5 Proof of Proposition 4

Because $(V - \mu_V, Y - \mathbb{E}Y, \tilde{\phi}_t - \mathbb{E}\tilde{\phi}_t)$ are jointly Gaussian, the best linear predictor of $V - \mu_V$ given $(Y, \tilde{\phi}_t)$ takes the form

$$\mathbb{E}[V - \mu_V \mid Y, \tilde{\phi}_t] = a_V\big(Y - \mathbb{E}Y\big) + a_\phi\big(\tilde{\phi}_t - \mathbb{E}\tilde{\phi}_t\big),$$

with (a_V, a_ϕ) determined by the multivariate regression system

$$\begin{pmatrix} \mathrm{Var}(Y) & \mathrm{Cov}(Y, \tilde{\phi}_t) \\ \mathrm{Cov}(Y, \tilde{\phi}_t) & \mathrm{Var}(\tilde{\phi}_t) \end{pmatrix} \begin{pmatrix} a_V \\ a_\phi \end{pmatrix} = \begin{pmatrix} \mathrm{Cov}(V, Y) \\ \mathrm{Cov}(V, \tilde{\phi}_t) \end{pmatrix}.$$

Because x_B depends on ϕ_t and $\tilde{\phi}_t = \phi_t + \xi_t$, we have $\mathrm{Cov}(Y, \tilde{\phi}_t) \neq 0$ generically, and therefore $a_\phi \neq 0$. This implies that equilibrium prices load directly on the noisy sentiment signal, as stated.

To show incomplete revelation, consider the conditional variance of ϕ_t given $(Y, \tilde{\phi}_t)$:

$$\mathrm{Var}(\phi_t \mid Y, \tilde{\phi}_t) = \sigma_\phi^2 - \mathrm{Cov}(\phi_t, Z)\Sigma_Z^{-1}\mathrm{Cov}(Z, \phi_t)',$$

where $Z = (Y, \tilde{\phi}_t)$ and Σ_Z is the covariance matrix of Z. Unless $\sigma_\xi^2 = 0$ and the mapping from ϕ_t to x_B is injective, Σ_Z is non-degenerate and the projective part cannot exhaust the variance of ϕ_t, so $\mathrm{Var}(\phi_t \mid Y, \tilde{\phi}_t)$ remains strictly positive.

In other words, because $(\phi_t, Y, \tilde{\phi}_t)$ are jointly Gaussian and both Y and $\tilde{\phi}_t$ contain independent noise components, the conditional distribution of ϕ_t given $(Y, \tilde{\phi}_t)$ remains Gaussian with variance strictly between zero and σ_ϕ^2 whenever $\sigma_\xi^2 > 0$. In particular, there is no choice of pricing rule that can eliminate this residual uncertainty, so sentiment is only partially revealed in equilibrium. □

A.6 Proof of Proposition 5

With the normalization $R_f = 0$ adopted in the main text, the one-period expected excess return satisfies the standard SDF pricing relation

$$\mathbb{E}[R_{t+1} \mid \phi_t] = -\frac{\mathrm{Cov}(M_{t+1}, R_{t+1} \mid \phi_t)}{\mathbb{E}[M_{t+1} \mid \phi_t]}.$$

Substituting the SDF,

$$M_{t+1} = m_0 - m_1 \Delta\phi_{t+1} - \sum_k m_k F_{k,t+1},$$

we obtain

$$\mathrm{Cov}(M_{t+1}, R_{t+1} \mid \phi_t) = -m_1 \mathrm{Cov}(\Delta\phi_{t+1}, R_{t+1} \mid \phi_t) - \sum_k m_k \mathrm{Cov}(F_{k,t+1}, R_{t+1} \mid \phi_t).$$

Define the factor betas

$$\beta_{\phi,t} = \frac{\mathrm{Cov}(R_{t+1}, \Delta\phi_{t+1} \mid \phi_t)}{\mathrm{Var}(\Delta\phi_{t+1})}, \qquad \beta_{k,t} = \frac{\mathrm{Cov}(R_{t+1}, F_{k,t+1} \mid \phi_t)}{\mathrm{Var}(F_{k,t+1})}.$$

Then

$$\mathrm{Cov}(M_{t+1}, R_{t+1} \mid \phi_t) = -m_1 \beta_{\phi,t} \mathrm{Var}(\Delta\phi_{t+1}) - \sum_k m_k \beta_{k,t} \mathrm{Var}(F_{k,t+1}),$$

so that

$$\mathbb{E}[R_{t+1} \mid \phi_t] = \frac{m_1 \beta_{\phi,t} \mathrm{Var}(\Delta\phi_{t+1})}{\mathbb{E}[M_{t+1}]} + \sum_k \frac{m_k \beta_{k,t} \mathrm{Var}(F_{k,t+1})}{\mathbb{E}[M_{t+1}]}.$$

Defining κ_ϕ and κ_k as in the statement yields the claimed representation. The strict positivity of $\beta_{\phi,t}$ follows from the dependence of returns on ϕ_t through $\lambda^*(\phi_t)$ and the sentiment-dependent noise in order flow. □

A.7 Proof of Proposition 6

The proof mirrors the single-asset case, but we keep explicit track of the vector structure of sentiment and returns.

Let R_{t+1} denote the $n \times 1$ vector of excess returns on the n sovereign bonds between t and $t+1$. The political component of the spread on bond i is

$$s_i(\phi_t) = \mathbb{E}[R_{i,t+1} \mid \phi_t] = -\frac{\mathrm{Cov}(M_{t+1}, R_{i,t+1} \mid \phi_t)}{\mathbb{E}[M_{t+1} \mid \phi_t]},$$

where we have normalized the risk-free rate to zero and used the standard Euler-equation relationship between expected excess returns and the covariance with the SDF. Because M_{t+1} is affine in the factors and $(R_{t+1}, \Delta\phi_{t+1}, F_{1,t+1}, \ldots, F_{L,t+1})$ are jointly Gaussian conditional on ϕ_t, all relevant expectations and covariances are well defined.

Step 1: Covariance of the SDF with returns. By assumption, the SDF has the form

$$M_{t+1} = m_0 - b'\Delta\phi_{t+1} - \sum_{\ell=1}^{L} c_\ell F_{\ell,t+1},$$

where b is a $K \times 1$ vector and the $F_{\ell,t+1}$ are scalar factors. For each bond i we have

$$\begin{aligned}\mathrm{Cov}(M_{t+1}, R_{i,t+1} \mid \phi_t) &= \mathrm{Cov}\left(-b'\Delta\phi_{t+1} - \sum_{\ell=1}^{L} c_\ell F_{\ell,t+1},\ R_{i,t+1} \mid \phi_t\right) \\ &= -b'\mathrm{Cov}(\Delta\phi_{t+1}, R_{i,t+1} \mid \phi_t) - \sum_{\ell=1}^{L} c_\ell \mathrm{Cov}(F_{\ell,t+1}, R_{i,t+1} \mid \phi_t).\end{aligned}$$

Since $\Delta\phi_{t+1} = \phi_{t+1} - \phi_t$ and ϕ_t is known at time t, we have $\mathrm{Var}(\Delta\phi_{t+1}) = \mathrm{Var}(\zeta_{t+1}) = \Sigma_\zeta$. Define the $K \times 1$ vector of sentiment betas for bond i as

$$\beta_{\phi,i,t} \equiv \Sigma_\eta^{-1} \mathrm{Cov}(R_{i,t+1}, \Delta\phi_{t+1} \mid \phi_t).$$

Equivalently,

$$\mathrm{Cov}(\Delta\phi_{t+1}, R_{i,t+1} \mid \phi_t) = \Sigma_\eta \beta_{\phi,i,t}.$$

Similarly, define the scalar betas with respect to the non-sentiment factors as

$$\beta_{\ell,i,t} \equiv \frac{\mathrm{Cov}(R_{i,t+1}, F_{\ell,t+1} \mid \phi_t)}{\mathrm{Var}(F_{\ell,t+1})}, \qquad \ell = 1, \ldots, L.$$

Substituting these definitions into the covariance expression, we obtain

$$\mathrm{Cov}(M_{t+1}, R_{i,t+1} \mid \phi_t) = -b'\Sigma_\eta \beta_{\phi,i,t} - \sum_{\ell=1}^{L} c_\ell \beta_{\ell,i,t} \mathrm{Var}(F_{\ell,t+1}). \tag{24}$$

Step 2: Expected excess returns and spreads. The conditional expectation of the SDF is

$$\mathbb{E}[M_{t+1} \mid \phi_t] = m_0 - b'\mathbb{E}[\Delta\phi_{t+1} \mid \phi_t] - \sum_{\ell=1}^{L} c_\ell \mathbb{E}[F_{\ell,t+1} \mid \phi_t].$$

Under the maintained assumption that the shocks have zero conditional mean, this reduces to a constant $\mathbb{E}[M_{t+1}]$ independent of ϕ_t, so we suppress the conditioning.

By the standard Euler-equation argument,

$$s_i(\phi_t) = \mathbb{E}[R_{i,t+1} \mid \phi_t] = -\frac{\mathrm{Cov}(M_{t+1}, R_{i,t+1} \mid \phi_t)}{\mathbb{E}[M_{t+1}]}.$$

Substituting (24) into this expression yields

$$s_i(\phi_t) = \frac{b'\Sigma_\eta \beta_{\phi,i,t}}{\mathbb{E}[M_{t+1}]} + \sum_{\ell=1}^{L} \frac{c_\ell \beta_{\ell,i,t} \mathrm{Var}(F_{\ell,t+1})}{\mathbb{E}[M_{t+1}]}.$$

Define the $K \times 1$ vector

$$\kappa_\phi \equiv \frac{\Sigma_\eta b}{\mathbb{E}[M_{t+1}]},$$

and the scalars

$$\kappa_\ell \equiv \frac{c_\ell \mathrm{Var}(F_{\ell,t+1})}{\mathbb{E}[M_{t+1}]}, \qquad \ell = 1, \ldots, L.$$

Then we can rewrite $b'\Sigma_\eta \beta_{\phi,i,t} = \kappa'_\phi \beta_{\phi,i,t}$ and obtain the factor-representation

$$s_i(\phi_t) = \kappa'_\phi \beta_{\phi,i,t} + \sum_{\ell=1}^{L} \kappa_\ell \beta_{\ell,i,t},$$

which is precisely (12).

Step 3: Positivity of sentiment-beta components. The sign statement in the last part of the proposition follows the same logic as in the single-asset case. In the multi-asset Kyle equilibrium, the vector of returns R_{t+1} depends on ϕ_t through the matrix price-impact coefficient

$\Lambda(\phi_t)$ and through the variance and composition of order flow, which in turn depend on the factor loadings B and the sentiment-dependent variance of biased order flow.

If, for some sentiment component k and bond i, the derivative $\partial\Lambda(\phi_t)/\partial\phi_{k,t}$ is non-zero and bond i loads on the corresponding order-flow component, the return $R_{i,t+1}$ will inherit a strictly positive covariance with $\Delta\phi_{k,t+1}$ conditional on ϕ_t. Given the Gaussian structure of ζ_{t+1} and the definition of $\beta_{\phi,i,t}$, this implies that the k-th element of $\beta_{\phi,i,t}$ is strictly positive.

More formally, let e_k denote the k-th unit vector in $\mathbb{R}^K$ and consider the decomposition $\Delta\phi_{t+1} = \zeta_{t+1} = \sum_{k=1}^{K} e_k\zeta_{k,t+1}$, with $\zeta_{k,t+1}$ the k-th component. If $R_{i,t+1}$ depends linearly on $\zeta_{k,t+1}$ with non-zero coefficient, then

$$\mathrm{Cov}(R_{i,t+1}, \zeta_{k,t+1} \mid \phi_t) \neq 0,$$

and the k-th element of $\mathrm{Cov}(R_{i,t+1}, \Delta\phi_{t+1} \mid \phi_t)$ is non-zero. Since Σ_η is positive definite, the inversion $\beta_{\phi,i,t} = \Sigma_\eta^{-1}\mathrm{Cov}(R_{i,t+1}, \Delta\phi_{t+1} \mid \phi_t)$ preserves strict positivity of that entry.

This completes the proof of the multi-country sentiment-beta representation and the positivity of the relevant components of $\beta_{\phi,i,t}$ under the Kyle equilibrium. □

A.8 Proof of Proposition 7

Using the linear equilibrium result, write the price as

$$P_t = \mu_V + \lambda^*(\phi_t)\big(Y_t - \mathbb{E}[Y_t \mid \phi_t]\big),$$

and recall from Lemma 1 that

$$Y_t - \mathbb{E}[Y_t \mid \phi_t] = \alpha(\phi_t)(V - \mu_V) + \eta(\phi_t),$$

where

$$\alpha(\phi_t) = \frac{\kappa_I}{1 + \kappa_I\lambda^*(\phi_t)}, \qquad \eta(\phi_t) = \frac{u_B(\phi_t) + x_L}{1 + \kappa_I\lambda^*(\phi_t)}.$$

The variables $V - \mu_V$ and $\eta(\phi_t)$ are jointly Gaussian with

$$\mathbb{E}[V - \mu_V \mid \phi_t] = 0, \quad \mathbb{E}[\eta(\phi_t) \mid \phi_t] = 0, \quad \mathrm{Cov}(V - \mu_V, \eta(\phi_t) \mid \phi_t) = 0,$$

and

$$\mathrm{Var}(V - \mu_V) = \sigma_V^2, \qquad \mathrm{Var}(\eta(\phi_t)) = \sigma_\eta^2(\phi_t).$$

Substituting the decomposition of order flow into the price equation gives

$$P_t - \mu_V = \lambda^*(\phi_t)\alpha(\phi_t)(V - \mu_V) + \lambda^*(\phi_t)\eta(\phi_t).$$

Step 1: Covariance between price and fundamentals. We have

$$\mathrm{Cov}(P_t, V \mid \phi_t) = \mathrm{Cov}\big(P_t - \mu_V,\ V - \mu_V \mid \phi_t\big).$$

Using the above expression for $P_t - \mu_V$,

$$\begin{aligned}\mathrm{Cov}(P_t, V \mid \phi_t) &= \mathrm{Cov}\Big(\lambda^*(\phi_t)\alpha(\phi_t)(V-\mu_V) + \lambda^*(\phi_t)\eta(\phi_t),\ V-\mu_V \ \Big|\ \phi_t\Big) \\ &= \lambda^*(\phi_t)\alpha(\phi_t)\,\mathrm{Var}(V-\mu_V) + \lambda^*(\phi_t)\,\mathrm{Cov}(\eta(\phi_t), V-\mu_V \mid \phi_t).\end{aligned}$$

By construction, $\mathrm{Cov}(\eta(\phi_t), V-\mu_V \mid \phi_t) = 0$, so

$$\mathrm{Cov}(P_t, V \mid \phi_t) = \lambda^*(\phi_t)\alpha(\phi_t)\sigma_V^2.$$

Step 2: Variance of price. We compute

$$\mathrm{Var}(P_t \mid \phi_t) = \mathrm{Var}(P_t - \mu_V \mid \phi_t).$$

Using the representation above,

$$\begin{aligned}\mathrm{Var}(P_t \mid \phi_t) &= \mathrm{Var}\Big(\lambda^*(\phi_t)\alpha(\phi_t)(V-\mu_V) + \lambda^*(\phi_t)\eta(\phi_t) \ \Big|\ \phi_t\Big) \\ &= \lambda^{*2}(\phi_t)\,\mathrm{Var}\big(\alpha(\phi_t)(V-\mu_V) + \eta(\phi_t) \mid \phi_t\big).\end{aligned}$$

Because $(V-\mu_V, \eta(\phi_t))$ are independent conditional on ϕ_t,

$$\mathrm{Var}\big(\alpha(\phi_t)(V-\mu_V) + \eta(\phi_t) \mid \phi_t\big) = \alpha(\phi_t)^2\sigma_V^2 + \sigma_\eta^2(\phi_t),$$

so

$$\mathrm{Var}(P_t \mid \phi_t) = \lambda^{*2}(\phi_t)\big(\alpha(\phi_t)^2\sigma_V^2 + \sigma_\eta^2(\phi_t)\big).$$

Step 3: The $R^2(\phi_t)$ expression and its monotonicity. By definition, the R^2 of price with respect to fundamentals, conditional on ϕ_t, is

$$R^2(\phi_t) \equiv \frac{\mathrm{Cov}(P_t, V \mid \phi_t)^2}{\mathrm{Var}(P_t \mid \phi_t)\,\mathrm{Var}(V)}.$$

Substituting the expressions above,

$$R^2(\phi_t) = \frac{\lambda^{*2}(\phi_t)\alpha(\phi_t)^2\sigma_V^4}{\lambda^{*2}(\phi_t)\big(\alpha(\phi_t)^2\sigma_V^2 + \sigma_\eta^2(\phi_t)\big)\sigma_V^2}.$$

Cancel the positive common factors $\lambda^{*2}(\phi_t)\sigma_V^2$ to obtain

$$R^2(\phi_t) = \frac{\alpha(\phi_t)^2\sigma_V^2}{\alpha(\phi_t)^2\sigma_V^2 + \sigma_\eta^2(\phi_t)}.$$

Define

$$N(\phi_t) \equiv \alpha(\phi_t)^2\sigma_V^2, \qquad D(\phi_t) \equiv \alpha(\phi_t)^2\sigma_V^2 + \sigma_\eta^2(\phi_t),$$

so that

$$R^2(\phi_t) = \frac{N(\phi_t)}{D(\phi_t)}.$$

From Lemma 1 and Assumption 2: - $\sigma_\eta^2(\phi_t)$ is strictly increasing in ϕ_t^2 (Proposition 2(i)), - $\lambda^*(\phi_t)$ is strictly decreasing in ϕ_t^2, - hence $\alpha(\phi_t) = \kappa_I/(1+\kappa_I\lambda^*(\phi_t))$ is bounded and (weakly) decreasing in ϕ_t^2.

Thus, as $|\phi_t|$ increases: - the numerator $N(\phi_t) = \alpha(\phi_t)^2\sigma_V^2$ is non-increasing, - the denominator $D(\phi_t) = N(\phi_t) + \sigma_\eta^2(\phi_t)$ increases strictly, because $\sigma_\eta^2(\phi_t)$ increases and eventually dominates.

Differentiating $R^2(\phi_t) = N(\phi_t)/D(\phi_t)$ with respect to ϕ_t^2 yields

$$\frac{\partial R^2(\phi_t)}{\partial(\phi_t^2)} = \frac{N'(\phi_t)D(\phi_t) - N(\phi_t)D'(\phi_t)}{D(\phi_t)^2}.$$

Here $D'(\phi_t) > 0$ and $N'(\phi_t) \le 0$, while $N(\phi_t) \ge 0$ and $D(\phi_t) > 0$. Therefore the numerator is strictly negative once $\sigma_\eta^2(\phi_t)$ starts rising, and

$$\frac{\partial R^2(\phi_t)}{\partial(\phi_t^2)} < 0.$$

Hence $R^2(\phi_t)$ is strictly decreasing in $|\phi_t|$, which proves the proposition. □

A.9 Proof of Proposition 8

We fix ϕ_t and work conditionally on ϕ_t. Let sentiment follow the AR(1) process

$$\phi_{t+1} = \rho\phi_t + \zeta_{t+1}, \qquad \zeta_{t+1} \sim \mathcal{N}(0, \sigma_\zeta^2), \quad |\rho| < 1,$$

and define the one-period sentiment innovation

$$\Delta\phi_{t+1} \equiv \phi_{t+1} - \phi_t = (\rho - 1)\phi_t + \zeta_{t+1}.$$

Conditional on ϕ_t, the random part of $\Delta\phi_{t+1}$ is ζ_{t+1}, and

$$\mathrm{Var}(\Delta\phi_{t+1} \mid \phi_t) = \sigma_\zeta^2.$$

From Proposition 5, the sentiment beta is

$$\beta_{\phi,t} = \frac{\mathrm{Cov}(R_{t+1}, \Delta\phi_{t+1} \mid \phi_t)}{\mathrm{Var}(\Delta\phi_{t+1} \mid \phi_t)} = \frac{\mathrm{Cov}(R_{t+1}, \Delta\phi_{t+1} \mid \phi_t)}{\sigma_\zeta^2}.$$

Hence

$$\frac{\partial\beta_{\phi,t}}{\partial\phi_t} = \frac{1}{\sigma_\zeta^2}\frac{\partial}{\partial\phi_t}\mathrm{Cov}(R_{t+1}, \Delta\phi_{t+1} \mid \phi_t). \tag{25}$$

Step 1: Affine representation of R_{t+1} in $\Delta\phi_{t+1}$. Under the linear-Gaussian Kyle block, and given Assumption 2, returns and sentiment innovations are jointly Gaussian conditional on ϕ_t. Therefore, the conditional regression of R_{t+1} on $\Delta\phi_{t+1}$ takes the affine form

$$R_{t+1} = a(\phi_t) + b(\phi_t)\,\Delta\phi_{t+1} + \varepsilon_{t+1}, \tag{26}$$

where ε_{t+1} is conditionally mean-zero and orthogonal to $\Delta\phi_{t+1}$ given ϕ_t:

$$\mathbb{E}[\varepsilon_{t+1} \mid \phi_t] = 0, \qquad \mathrm{Cov}(\varepsilon_{t+1}, \Delta\phi_{t+1} \mid \phi_t) = 0.$$

By definition of the regression coefficient,

$$b(\phi_t) = \frac{\mathrm{Cov}(R_{t+1}, \Delta\phi_{t+1} \mid \phi_t)}{\mathrm{Var}(\Delta\phi_{t+1} \mid \phi_t)} = \frac{\mathrm{Cov}(R_{t+1}, \Delta\phi_{t+1} \mid \phi_t)}{\sigma_\zeta^2}. \tag{27}$$

In particular,

$$\mathrm{Cov}(R_{t+1}, \Delta\phi_{t+1} \mid \phi_t) = b(\phi_t)\,\sigma_\zeta^2. \tag{28}$$

Moreover, since ε_{t+1} is orthogonal to $\Delta\phi_{t+1}$, the conditional variance of R_{t+1} can be decomposed as

$$\mathrm{Var}(R_{t+1} \mid \phi_t) = b(\phi_t)^2\,\mathrm{Var}(\Delta\phi_{t+1} \mid \phi_t) + \mathrm{Var}(\varepsilon_{t+1} \mid \phi_t) = b(\phi_t)^2\sigma_\zeta^2 + \sigma_\varepsilon^2(\phi_t), \tag{29}$$

where $\sigma_\varepsilon^2(\phi_t) \equiv \mathrm{Var}(\varepsilon_{t+1} \mid \phi_t)$.

Step 2: Linking $b(\phi_t)$ to $\mathrm{Var}(R_{t+1} \mid \phi_t)$. From Proposition 2, $\mathrm{Var}(R_{t+1} \mid \phi_t)$ is strictly increasing in $|\phi_t|$ through the dependence of the composite noise variance $\sigma_\eta^2(\phi_t)$ on $\sigma_B^2(\phi_t)$.

To obtain a clean relationship between the regression slope $b(\phi_t)$ and $\mathrm{Var}(R_{t+1} \mid \phi_t)$, we impose the following mild regularity condition: for each ϕ_t, the component of R_{t+1} that is orthogonal to $\Delta\phi_{t+1}$ has conditional variance that is weakly decreasing in $|\phi_t|$, i.e.

$$\frac{\partial}{\partial|\phi_t|}\sigma_\varepsilon^2(\phi_t) \le 0. \tag{30}$$

This is natural in our Kyle setting: as the contribution of sentiment-driven noise to order flow grows, a larger fraction of the conditional variance of returns is explained by movements in $\Delta\phi_{t+1}$ rather than by idiosyncratic shocks.

Differentiating (29) with respect to $|\phi_t|$ yields

$$\frac{\partial}{\partial|\phi_t|}\mathrm{Var}(R_{t+1} \mid \phi_t) = 2b(\phi_t)\sigma_\zeta^2\frac{\partial b(\phi_t)}{\partial|\phi_t|} + \frac{\partial\sigma_\varepsilon^2(\phi_t)}{\partial|\phi_t|}.$$

Proposition 2 implies that the left-hand side is strictly positive, and (30) implies that the last term on the right-hand side is weakly non-positive. Hence the term $2b(\phi_t)\sigma_\zeta^2\partial b(\phi_t)/\partial|\phi_t|$ must be strictly positive, so

$$b(\phi_t)\,\frac{\partial b(\phi_t)}{\partial|\phi_t|} > 0 \qquad \text{for all } \phi_t \neq 0. \tag{31}$$

In particular, $b(\phi_t)$ and $\partial b(\phi_t)/\partial|\phi_t|$ have the same sign. Because stronger sentiment makes biased order flow more volatile, the regression coefficient $b(\phi_t)$ cannot be zero for $\phi_t \neq 0$, so (31) implies

$$\frac{\partial b(\phi_t)}{\partial|\phi_t|} > 0 \qquad \text{for all } \phi_t \neq 0. \tag{32}$$

Step 3: Monotonicity of $\beta_{\phi,t}$ and $s(\phi_t)$. Combining (28) and (32),

$$\frac{\partial}{\partial|\phi_t|}\mathrm{Cov}(R_{t+1}, \Delta\phi_{t+1} \mid \phi_t) = \sigma_\zeta^2 \frac{\partial b(\phi_t)}{\partial|\phi_t|} > 0 \qquad \text{for all } \phi_t \neq 0.$$

Using (25) and the fact that $\mathrm{Var}(\Delta\phi_{t+1} \mid \phi_t) = \sigma_\zeta^2$ does not depend on ϕ_t, we conclude that

$$\frac{\partial \beta_{\phi,t}}{\partial|\phi_t|} = \frac{1}{\sigma_\zeta^2}\frac{\partial}{\partial|\phi_t|}\mathrm{Cov}(R_{t+1}, \Delta\phi_{t+1} \mid \phi_t) > 0 \qquad \text{for all } \phi_t \neq 0.$$

Finally, since $s(\phi_t) = \kappa_\phi \beta_{\phi,t}$ with $\kappa_\phi > 0$, we obtain

$$\frac{\partial s(\phi_t)}{\partial|\phi_t|} = \kappa_\phi \frac{\partial \beta_{\phi,t}}{\partial|\phi_t|} > 0 \qquad \text{for all } \phi_t \neq 0,$$

which proves the proposition. □

A.10 Proof of Proposition 9

Recall the definition of the net advantage of confrontation over concession:

$$D(\phi) = U_G(CF, \phi) - U_G(CC, \phi) = \big[R(CF) - R(CC)\big] - \big[C_{CF}(\phi) - C_{CC}(\phi)\big] - D_{CF} \times s(\phi),$$

where: - $R(CF) - R(CC)$ is a constant geopolitical benefit differential, - $C_{CF}(\phi)$ and $C_{CC}(\phi)$ are political cost functions satisfying Assumption 4, - $s(\phi)$ is the sovereign spread, strictly increasing in $|\phi|$ by Propositions 2 and 8.

By the assumptions of Proposition 9: (i) $D(\phi)$ is continuous in ϕ. (ii) There exists $\phi^\star$ such that $D(\phi^\star) > 0$. (iii) $\lim_{\phi\to\infty} D(\phi) < 0$.

Existence of a threshold. Take $\phi^\star$ with $D(\phi^\star) > 0$. By (iii), there exists Φ such that $D(\phi) < 0$ for all $\phi > \Phi$. By continuity of D on $[\phi^\star, \Phi]$, the Intermediate Value Theorem guarantees the existence of at least one $\bar{\phi} \in [\phi^\star, \Phi]$ such that

$$D(\bar{\phi}) = 0.$$

Monotonicity for large ϕ. For sufficiently large ϕ, Assumption 4 implies that

$$\frac{\mathrm{d}}{\mathrm{d}\phi}\big[C_{CF}(\phi) - C_{CC}(\phi)\big] > 0,$$

and Propositions 2 and 8 imply that

$$\frac{\mathrm{d}s(\phi)}{\mathrm{d}\phi} > 0.$$

Thus, for large ϕ,

$$D'(\phi) = -\big[C'_{CF}(\phi) - C'_{CC}(\phi)\big] - D_{CF}s'(\phi).$$

Both terms on the right-hand side are strictly negative (since $D_{CF} > 0$), so there exists ϕ_0 such that

$$D'(\phi) < 0 \quad \text{for all } \phi \geq \phi_0.$$

Hence D is strictly decreasing on $[\phi_0, \infty)$. We can choose $\phi_0 \leq \bar{\phi}$ without loss of generality.

Uniqueness of the threshold. Suppose, by contradiction, there exist two thresholds $\bar{\phi}_1 < \bar{\phi}_2$ with

$$D(\bar{\phi}_1) = D(\bar{\phi}_2) = 0.$$

Because $D'(\phi) < 0$ for $\phi \geq \phi_0$ and both $\bar{\phi}_1, \bar{\phi}_2 \geq \phi_0$, strict monotonicity implies

$$D(\bar{\phi}_2) < D(\bar{\phi}_1),$$

contradicting $D(\bar{\phi}_1) = D(\bar{\phi}_2)$. Hence the solution $\bar{\phi}$ to $D(\bar{\phi}) = 0$ is unique.

Sign of $D(\phi)$ around the threshold. Because $D(\phi^\star) > 0$ for some $\phi^\star < \bar{\phi}$ and $\lim_{\phi\to\infty} D(\phi) < 0$, and D is continuous and strictly decreasing for $\phi \geq \phi_0$, the unique zero $\bar{\phi}$ must be a crossing point from positive to negative values. Thus:

$$\phi_t < \bar{\phi} \Rightarrow D(\phi_t) > 0 \quad \text{(confrontation preferred)},$$

$$\phi_t > \bar{\phi} \Rightarrow D(\phi_t) < 0 \quad \text{(concession preferred)}.$$

This completes the proof and justifies calling $\bar{\phi}$ the sentiment constraint threshold. □

A.11 Proof of Proposition 10

Under the dynamic extension, sentiment evolves according to

$$\phi_{t+1} = \begin{cases} \phi_t + \Delta_{CF} + \eta' & \text{if } a_t = CF, \\ \phi_t + \Delta_{CC} + \eta' & \text{if } a_t = CC, \end{cases}$$

with $\Delta_{CF} > \Delta_{CC}$ capturing the idea that confrontation aggravates polarization while concession mitigates it, and η' a mean-zero shock.

The government's Bellman equation is

$$V_G(\phi) = \max_{a \in \{CF, CC\}} \big\{ U_G(a, \phi) + \delta \mathbb{E}[V_G(\phi') \mid a, \phi] \big\},$$

where $U_G(a, \phi)$ is the per-period payoff and $\delta \in (0, 1)$ is the discount factor.

Define the dynamic advantage of confrontation as

$$\begin{aligned} \Delta W(\phi) &\equiv W_G(CF, \phi) - W_G(CC, \phi) \\ &= \Big[U_G(CF, \phi) + \delta \mathbb{E}[V_G(\phi'_{CF}) \mid \phi] \Big] - \Big[U_G(CC, \phi) + \delta \mathbb{E}[V_G(\phi'_{CC}) \mid \phi] \Big] \\ &= D(\phi) + \delta \Big(\mathbb{E}[V_G(\phi + \Delta_{CF} + \eta') \mid \phi] - \mathbb{E}[V_G(\phi + \Delta_{CC} + \eta') \mid \phi] \Big), \end{aligned}$$

where $D(\phi) = U_G(CF, \phi) - U_G(CC, \phi)$ is the static advantage from Proposition 9.

Assume $V_G(\cdot)$ is differentiable and strictly decreasing in ϕ (higher sentiment reduces continuation value). Using the mean-value theorem inside the expectation, there exists, for each realization of η', a point $\tilde{\phi}(\eta')$ between $\phi + \Delta_{CC} + \eta'$ and $\phi + \Delta_{CF} + \eta'$ such that

$$V_G(\phi + \Delta_{CF} + \eta') - V_G(\phi + \Delta_{CC} + \eta') = V'_G(\tilde{\phi}(\eta'))(\Delta_{CF} - \Delta_{CC}).$$

Taking expectations conditional on ϕ,

$$\mathbb{E}\big[V_G(\phi + \Delta_{CF} + \eta') - V_G(\phi + \Delta_{CC} + \eta') \mid \phi \big] = (\Delta_{CF} - \Delta_{CC})\, \mathbb{E}[V'_G(\tilde{\phi}(\eta')) \mid \phi].$$

Since $V'_G(\cdot) < 0$ and $\Delta_{CF} - \Delta_{CC} > 0$, the right-hand side is strictly negative, so

$$\mathbb{E}[V_G(\phi + \Delta_{CF} + \eta') \mid \phi] - \mathbb{E}[V_G(\phi + \Delta_{CC} + \eta') \mid \phi] < 0.$$

Therefore,

$$\Delta W(\phi) = D(\phi) + \delta \Big(\mathbb{E}[V_G(\phi + \Delta_{CF} + \eta')] - \mathbb{E}[V_G(\phi + \Delta_{CC} + \eta')] \Big) < D(\phi).$$

Part (i). Let $\bar{\phi}$ be the static threshold such that $D(\bar{\phi}) = 0$, and let $\bar{\phi}^{\text{dyn}}$ be defined by $\Delta W(\bar{\phi}^{\text{dyn}}) = 0$. Since $\Delta W(\phi) < D(\phi)$ for all ϕ, the dynamic advantage crosses zero earlier. In particular, for $\phi = \bar{\phi}$, we have $D(\bar{\phi}) = 0$ and $\Delta W(\bar{\phi}) < 0$. By continuity of $\Delta W(\cdot)$, there must exist $\bar{\phi}^{\text{dyn}} < \bar{\phi}$ such that $\Delta W(\bar{\phi}^{\text{dyn}}) = 0$. Thus

$$\bar{\phi}^{\text{dyn}} < \bar{\phi}.$$

Part (ii). The set of ϕ such that confrontation is optimal in the dynamic problem is

$$\{\phi : \Delta W(\phi) > 0\}.$$

Because $\Delta W(\phi) < D(\phi)$ for all ϕ, and $D(\phi)$ crosses zero at $\bar{\phi}$, it follows that

$$\{\phi : \Delta W(\phi) > 0\} \subset \{\phi : D(\phi) > 0\}.$$

Thus the range of sentiment values for which confrontation is optimal shrinks when the government is forward-looking. This proves the proposition. □

A.12 Proof of Proposition 11

Write $D(\phi; D_{CF})$ to emphasize the dependence on the additional borrowing requirement D_{CF}, and suppose $D(\phi; D_{CF})$ satisfies the conditions of Proposition 9 for all D_{CF} in an open interval around the baseline value. Let $\bar{\phi}(D_{CF})$ denote the unique solution to

$$D(\bar{\phi}(D_{CF}); D_{CF}) = 0.$$

Implicit differentiation with respect to D_{CF} yields

$$\left.\frac{\partial D}{\partial \phi}\right|_{\phi=\bar{\phi}} \frac{\mathrm{d}\bar{\phi}}{\mathrm{d}D_{CF}} + \left.\frac{\partial D}{\partial D_{CF}}\right|_{\phi=\bar{\phi}} = 0,$$

so

$$\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}D_{CF}} = -\left.\frac{\dfrac{\partial D}{\partial D_{CF}}}{\dfrac{\partial D}{\partial \phi}}\right|_{\phi=\bar{\phi}}.$$

From the definition

$$D(\phi; D_{CF}) = \big[R(CF) - R(CC)\big] - \big[C_{CF}(\phi) - C_{CC}(\phi)\big] - D_{CF}\, s(\phi),$$

we have

$$\frac{\partial D}{\partial D_{CF}} = -s(\phi), \qquad \frac{\partial D}{\partial \phi} = -\big(C'_{CF}(\phi) - C'_{CC}(\phi)\big) - D_{CF} s'(\phi).$$

Evaluating at $\phi = \bar{\phi}(D_{CF})$ gives

$$\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}D_{CF}} = -\frac{-s(\bar{\phi})}{-\big(C'_{CF}(\bar{\phi}) - C'_{CC}(\bar{\phi})\big) - D_{CF} s'(\bar{\phi})} = -\frac{s(\bar{\phi})}{C'_{CF}(\bar{\phi}) - C'_{CC}(\bar{\phi}) + D_{CF} s'(\bar{\phi})}.$$

By Assumption 4, $C'_{CF}(\cdot) - C'_{CC}(\cdot) > 0$ for large ϕ, and $s'(\phi) > 0$ by the microstructure results. Thus, the denominator is strictly positive at $\phi = \bar{\phi}(D_{CF})$, while the numerator $s(\bar{\phi}) > 0$. Hence

$$\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}D_{CF}} < 0.$$

An increase in the financing requirement D_{CF} shifts the constraint threshold downward and

reduces the range of sentiment levels for which confrontation is optimal, as claimed. □

A.13 Proof of Proposition 12

Throughout this proof we fix ϕ_t as a state and treat all variances and covariances as conditional on ϕ_t.

Under the assumptions of Proposition 9, the net advantage of confrontation when the delegated-portfolio share is θ can be written as

$$D(\phi;\theta) = \big[R(\mathrm{CF}) - R(\mathrm{CC})\big] - \big[C_{\mathrm{CF}}(\phi) - C_{\mathrm{CC}}(\phi)\big] - D_{CF} \times s(\phi;\theta),$$

where $s(\phi;\theta)$ is the spread function in an economy with delegated share θ.

For any fixed θ in the interval considered, $D(\phi;\theta)$ is continuous and strictly decreasing in ϕ on $[\phi_0, \infty)$, and the regularity conditions in Proposition 9 ensure the existence of a unique threshold $\bar{\phi}(\theta)$ solving

$$D(\bar{\phi}(\theta);\theta) = 0.$$

We now study how $\bar{\phi}(\theta)$ changes with θ. Applying the implicit function theorem to the identity $D(\bar{\phi}(\theta);\theta) = 0$ yields

$$\left.\frac{\partial D}{\partial \phi}\right|_{\phi=\bar{\phi}(\theta)} \frac{\mathrm{d}\bar{\phi}(\theta)}{\mathrm{d}\theta} + \left.\frac{\partial D}{\partial \theta}\right|_{\phi=\bar{\phi}(\theta)} = 0,$$

so that

$$\frac{\mathrm{d}\bar{\phi}(\theta)}{\mathrm{d}\theta} = -\left.\frac{\dfrac{\partial D}{\partial \theta}}{\dfrac{\partial D}{\partial \phi}}\right|_{\phi=\bar{\phi}(\theta)}.$$

From the definition of $D(\phi;\theta)$,

$$\frac{\partial D}{\partial \theta} = -D_{CF}\,\frac{\partial s(\phi;\theta)}{\partial \theta}, \qquad \frac{\partial D}{\partial \phi} = -\big(C'_{\mathrm{CF}}(\phi) - C'_{\mathrm{CC}}(\phi)\big) - D_{CF}\,\frac{\partial s(\phi;\theta)}{\partial \phi}.$$

Evaluating at $\phi = \bar{\phi}(\theta)$ gives

$$\frac{\mathrm{d}\bar{\phi}(\theta)}{\mathrm{d}\theta} = -\frac{D_{CF}\,\dfrac{\partial s(\bar{\phi}(\theta);\theta)}{\partial \theta}}{C'_{\mathrm{CF}}(\bar{\phi}(\theta)) - C'_{\mathrm{CC}}(\bar{\phi}(\theta)) + D_{CF}\,\dfrac{\partial s(\bar{\phi}(\theta);\theta)}{\partial \phi}}.$$

By Assumption 4 and the monotonicity of the spread in ϕ established in Proposition 2 (and Proposition 8 when needed), the denominator is strictly positive at $\phi = \bar{\phi}(\theta)$. Proposition 3 shows that, when $\sigma^2_{B,\mathrm{del}} > \sigma^2_{B,\mathrm{dir}}$, delegated portfolios increase the composite noise variance and

thereby raise the political component of the spread: for any fixed $\phi_t \neq 0$,

$$\frac{\partial s(\phi_t;\theta)}{\partial \theta} > 0.$$

Thus, $D_{CF}\,\dfrac{\partial s(\bar{\phi}(\theta);\theta)}{\partial \theta}$ is strictly positive for $\phi_t \neq 0$, so the fraction as a whole is positive both in the numerator and in the denominator, implying

$$\frac{\mathrm{d}\bar{\phi}(\theta)}{\mathrm{d}\theta} < 0.$$

Therefore, a higher delegated-portfolio share θ tightens the sentiment constraint by shifting the threshold $\bar{\phi}(\theta)$ downward: the range of sentiment levels for which confrontation is optimal shrinks as more of the sovereign debt is held through delegated portfolios with explicit identity tilts. This completes the proof. □